

A study on Modified Advanced First Order Second Moment method for Beam-to-Column Connections

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Abstract. Beam-to-column connections are involved in material and geometric uncertainties. In the deterministic structural analysis, it is difficult to estimate the proper safety factor of structures because the analysis does not consider these uncertainties. Structural reliability analysis is a more reasonable method because these uncertainties are considered by calculating means and standard deviations when determining the safety factor of structure. A three -parameter power model is used for defining the moment-rotation relationship for example for angle-type connections. For structural reliability analysis, probability distribution functions of three parameters treated as random variables are assumed to follow a normal distribution. The three parameters are ultimate moment M_u , initial rotational stiffness R_{ki} and shape parameter n . The reliability index and probabilities of failure at a certain M-O point are calculated by Advanced First Order Second Moment method.

Keywords: Angle type connection, Reliability, AFOSM(Advanced First Order Second Moment) Method

1 Introduction

One of the possibilities to improve the accuracy of steel structures analysis can be evaluation of the actual behaviour of joints. For these purposes the semi-rigid joint conception was introduced. The most relevant influence on the rigidity of the joint is produced by bending moments. In this case the behaviour of a semi-rigid joint is described by moment-rotation $M-\theta$ curve of the joint. The joints are also influenced by axial and shear forces. These forces usually are not evaluated in calculating the rigidity and strength of the joints.

For typical structural systems, failure is defined as the formation of an unstable collapse state under some applied loads. The objective of this study is to propose the reliability analysis which can be evaluated probability of failure and reliability index of steel frames with connections. Also the limit state equation which shows limit state

¹ Please note that the LNCS Editorial assumes that all authors have used the western naming convention, with given names preceding surnames. This determines the structure of the names in the running heads and the author index.

of steel frames is investigated. In this study, the moment-rotation relationship of connections is assumed by the power model that has three parameters such as initial stiffness, ultimate moment capacity and shape coefficient of the connection. In order to calculate the probability of failure and reliability index of steel connections, advanced first order second moment method and Monte-Carlo method using inverse transformation method are adopted. Strength limit state equation for reliability analysis of steel beam is presented. In the present study, assumption and scopes are limits as follows : All random variable are statistically independent to analysis more easily. Distributed types are assumed as standard normal distributions. Only effect by bending moment except for twisting are considered and bucklings of flange in column and web in beam are ignored. The methodologies which analyze reliability of beam-to-column connections under monotonic load are as follows. Firstly, the moment-rotation relationship of five types connections which are suggested by R. Hasan(1998) are assumed by the power model that have three parameters such as initial stiffness R_k , ultimate moment capacity M_u , and shape coefficient n of the connection. Secondly, limit state equations which are used by three-parameter power model by Kishi-Chen are suggested to perform the reliability analysis. It is done by AFOSM-Advanced First Order Second Moment Method. Beam-to-column connections are involved uncertainties of material and geometric properties. The established deterministic structural analysis method is difficult to estimate the proper safety because of no considering these uncertainties. The purpose of structural reliability analysis is to assess the safety of a structure more properly by considering its inherent uncertainties. In the method of structural reliability analysis, level technique is the most general method. In this study, the moment-rotation relation is adopted by power model considering three parameters. For analyzing structural reliability of angle-type connections, probability distribution functions of three parameters are the ultimate moment capacity M_u , the initial stiffness R_k , and the shape parameter. The reliability indices and probabilities of failure are calculated by Advanced First Order Second Moment method. With the ideas, in section 2 models of beam-to-column connection and procedure of reliability are introduced. In order to verify the proposed method numerical examples of connections are treated in section 3. Finally conclusions of this paper are discussed in section 4.

2 Analysis

2.1 Models of beam-to-column connection

2.1.1 Three parameter power models

The connection behavior is represented by its moment-rotation relationship. Herein, the three-parameter power model proposed by Richard(1961) is used and may be written as

$$M_u = \frac{R_k}{1 + \left(\frac{\theta}{\theta_0}\right)^n} \quad (1)$$

M_u ultimate moment capacity, R_{ki} initial stiffness, n shape parameter and θ_0 reference plastic rotation(= M_u / R_{ki}).

Table 1. Typical types of beam-to-column connections and model name

Models	Types
Type1	Single web-angle connection
Type2	Double web-angle connection
Type3	Top-and Seat-angle connection
Type4	Top-and Seat-angle connection with double web-angle
Type5	Header-plate connection

2.1.2 Ultimate moment capacity M_u

The mechanism moment capacity of a connection is reached when an idealized elastic-plastic collapse mechanism is developed in the assembly angles. On the basis of experimental studies, the collapse mechanism of a connection may be obtained by summation of the plastic moment capacities contributed by assembly angles. Herein, plastic beam theory considering the bending moment-shear interaction is used to derive the expression for the mechanism moment.

Table 2. Ultimate moment capacity according to types

Models	Types of connection
Type1	$M_u = \frac{2 V_p + V_0}{6} l_p^2$
Type2	single web-angle $\times 2$: $M_u = \frac{2 V_p + V_0}{3} l_p^2$
Type3	$M_u = M_{0s} + M_p + V_p d_2$
Type4	superposition double web-angle and top&seat-angle : $M_u = \frac{M_s}{6} + \frac{M_p}{6} + \frac{V_p d_2}{6} + \frac{2 V_p d_4}{6}$
Type5	similar behavior with double web-angle : $M_u = \frac{2 V_p + V_0}{3} l_p^2$

2.1.3 Initial stiffness R_{ki}

Based on these assumptions and considering shear deformation, the initial stiffness can be formulated from simple elastic theory.

2.1.4 Shape parameter n

Based on these assumptions and considering shear deformation, the initial stiffness can be formulated from simple elastic theory. The value of n is determined from

experimental tests by applying least mean square technique using equation. Numerical values of are then plotted against $\log_{10}(\dots)$. The shape parameter n is assumed to be a linear function of $\log_{10}(\dots)$ and is obtained by linear regression analysis from the $n \square \log_{10}(\dots)$ curve. To avoid negative values of n , a horizontal segment is added to the included line.

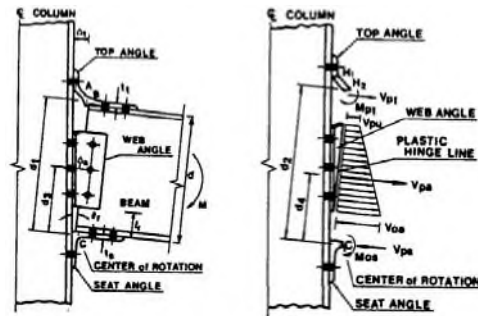


Fig. 1. Top and Seat angle with double web angle connection
 (a) Deflection configuration of elastic condition
 (b) Applied forces in ultimate state of the connection

Table 3. Initial Stiffness according to types

Models	Types of connection
Type1	Double web-angle $\times 1/2 : R_{k\bar{r}} \frac{3 EI_d (d)^3}{g_3 (g_3^2 + 0.78 \frac{t}{a})}$
Type2	Single web-angle $\times 2 : R_{k\bar{r}} \frac{6 ()^2 EI_d (d)^3}{g_3 (g_3^2 + 0.78 \frac{t}{a})^2}$
Type3	$R_{k\bar{r}} \frac{3 EI_d (d)^2}{g_3 (g_3^2 + 0.78 \frac{t}{a})}$
Type4	Superposition double web-angle and top & seat-angle $\frac{EI_d d}{g_3 (g_3^2 + 0.78 \frac{t}{a})} + \frac{2}{3 (g_3^2 + 0.78 \frac{t}{a})}$
Type5	$R_{k\bar{r}} \frac{6 EI_d (d)^2}{g_3 (g_3^2 + 0.78 \frac{t}{a})}$

) g

+

3 (

Similar behaviour with double web-angle

g_1



Table 4. Shape parameter according to types

Models	Shape parameter function	
Type1	$0.520 \log_{10}(\theta + 2.2910)$ 695	$\log_{10}(\theta) > \square 3.073$ $\log_{10}(\theta) \delta \square 3.073$
Type2	$1.332 \log_{10}(\theta + 3.952)$ 0.5371	$\log_{10}(\theta) > \square 2.582$ $\log_{10}(\theta) \delta \square 2.582$
Type3	$2.003 \log_{10}(\theta + 6.0700)$ 302	$\log_{10}(\theta) > \square 2.880$ $\log_{10}(\theta) \delta \square 2.880$
Type4	$1.398 \log_{10}(\theta + 4.631)$ 0.827	$\log_{10}(\theta) > \square 2.721$ $\log_{10}(\theta) \delta \square 2.721$
Type5	$0.512 \log_{10}(\theta + 2.2641)$.041	$\log_{10}(\theta) > \square 2.390$ $\log_{10}(\theta) \delta \square 2.390$

2.2 Reliability

Based on these assumptions and considering shear deformation, the initial stiffness can be formulated from simple elastic theory. To reliability analysis, AFOSM(Advanced First Order Second Moment) method is used. AFOSM method is one of Level II method. It makes the limit state equation first or second order function. And then by using probability density function it can calculate reliability index concerning with probability of failure. The proposed procedure for computing reliability index and probability of failure is shown in the Fig. 4. This proposed method is enough to include all angle types of beam-to-column connections.

The reliability index, β , is related to the probability of failure, P_F as

$$\beta = \sqrt{-\ln P_F} \quad (2)$$

Let R be resistance, Q be the load effect and limit state function

$$g(x) = R - Q \quad (3)$$

2.2.1 Limit states equation

Moment M is acting load and limit state determine plastic rotation capacity of Special Moment Frame that θ is 0.03 rad then limit state equation are proposed as follows.

$$g = \frac{1}{M_u} \left(\frac{R k_i}{k_l} \right)^{1/n} - \frac{M}{M_u} \quad (4)$$

2.2.2

Random variables

Random variables which use mean and standard deviation are applied to consider the uncertainties of each variable in the reliability theory.

Design variables which composite parameters are material properties, geometric dimension and load. Deviation of material and load variables are 5% of mean and Deviation of geometric dimension is 0.2% of mean which is calculated from reliability 99.8%. In material variable, elastic modulus E is 29000ksi, yielding strength is 36 ksi of SS41 steel and dimension-geometric variable- is applied by mean value of design variable with reference to AISC manual. Dimensions according to types of connection are given in Fig. 3, Table 5 and Table 6.

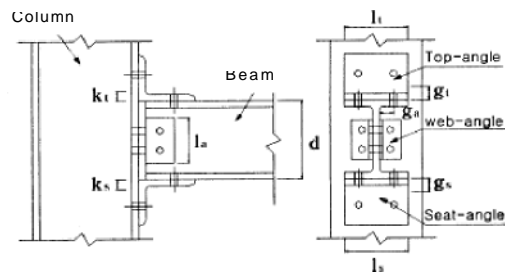


Fig.3. Dimension of analytical model

3 Numerical Example

To investigate the change of value, input the arbitrary moment and then increase the load to limit state about five types model that proposed in the former chapter.

3.1 Relation of behavior and reliability index according to types of connections

To investigate the change of value, input the arbitrary moment and then increase the load to limit state about five types model that proposed in the former chapter. The curve of $M-O-\beta$ has almost same form because the behaviour of angle-type connection is similar.

Table 4. Parameters for analysis

Type	Model	Angle	$L_p(l)$	$G_c(gt)$	$K_a(kt)$	W-Shape	d
1	Single web angle	L4 -3.5*1/4	5.5	2.56	11/16	WB 8*21	8
2	Double web angle	2L6 - 4*3/8	6	2.63	7/8	WB 8*21	8
3	Top and seat angle	L4 - 4*1/4*5.0	4	2.63	1/4	WB 5*16 WC 5*16	5 8
4	Top and seat angle with double web angle	2L4 - 3.5*1/4*5.5 L6 - 3.5*5/16*6.0	5.5 6	2.5 3.5	11/16 13/16	WB 8*21 WC 12*58	12 18
5	Header plate	-	15	4	1/4	WB 8*21	

However, the change of value is gentle as connection is close to pin connection. Fig.5. shows the curves of M-O- β according to type of connections. Using a reliability index and a probability of failure, which are obtained by the presented program, this thesis shows relations between a curve of moment-rotational angle and a reliability index at each type of connection parts as 3-dimensional curves, and it follows as a consequence that this paper presents a margin of safety and a degree of reinforcement and describe mutual relations of each connection part.

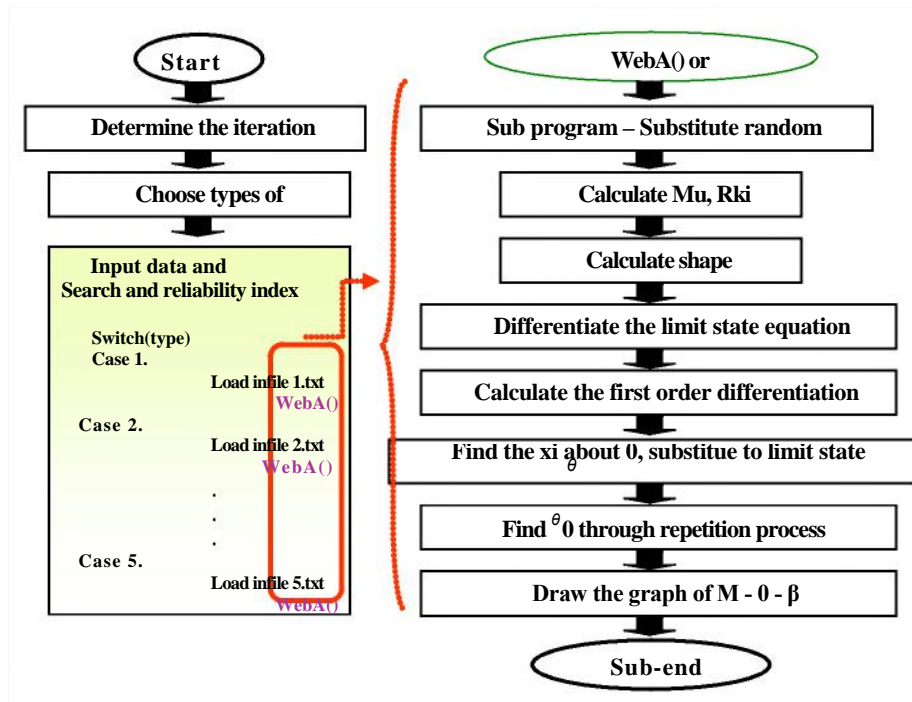
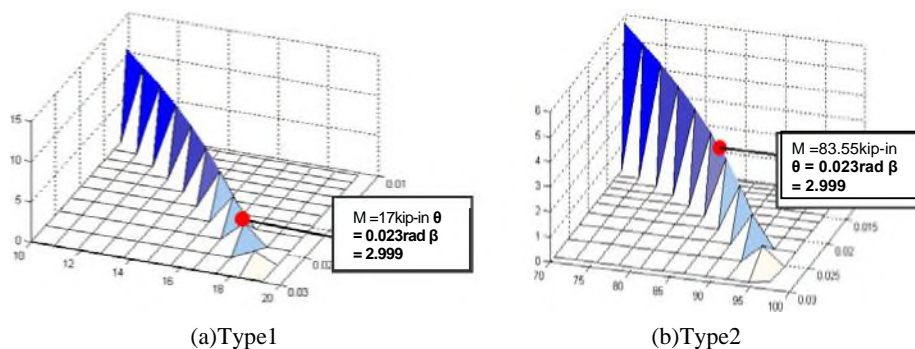


Fig.4. Reliability Program of connections



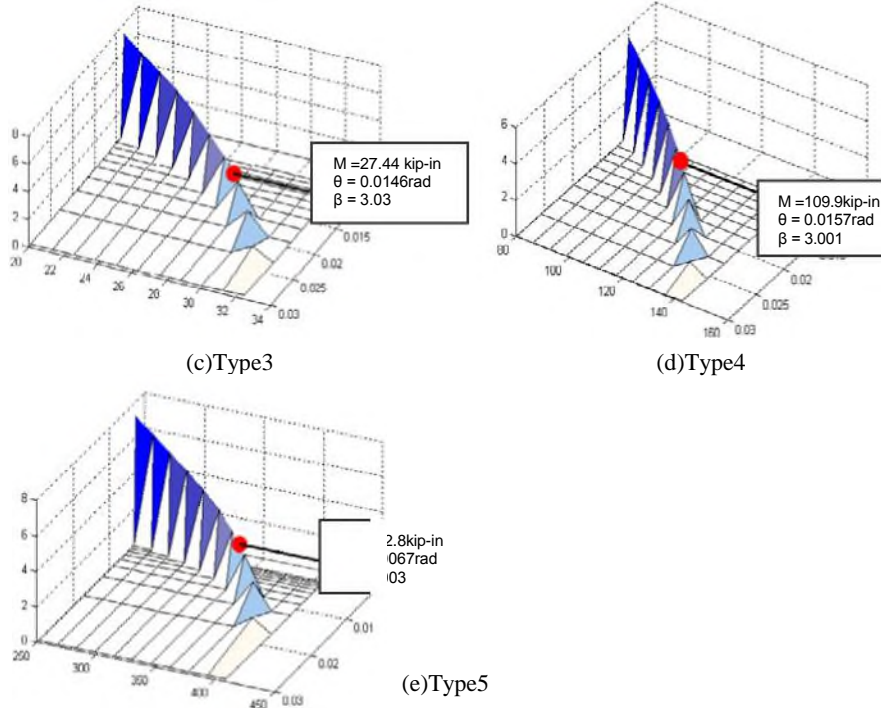


Fig. 5. Curve of M-0-13

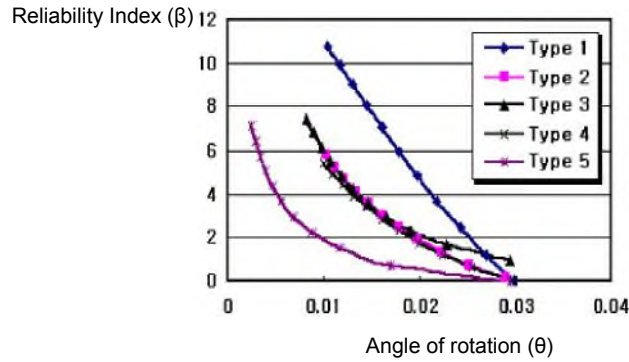


Fig. 6. Curve of 0-13

4 Conclusion

To investigate Beam-to-column connections are involved uncertainties of material and geometric properties. The established deterministic structural analysis method is

difficult to estimate the proper safety because of no considering these uncertainties. The purpose of structural reliability analysis is to assess the safety of a structure more properly by considering its inherent uncertainties. In the method of structural reliability analysis, level technique is the most general method. In this study, the moment-rotation relation is adopted by power model considering three parameters. For analyzing structural reliability of angle-type connections, probability distribution functions of three parameters treated as random variables are assumed to be normal distribution. Three parameters are the ultimate moment capacity M_u , the initial connection stiffness R_{ki} and the shape parameter n . The reliability indices and probabilities of failure are calculated by Advanced First-Order Second-Moment method. The accuracy of the present method is verified by Monte Carlo Simulation method, which is one of accurate methods to obtain failure probability. Analysis examples result in below lists.

1. With using past experiment data about beam-to-column connection, limit state equation dealing the moment-rotation relationship and extremely limit range was suggested by the way for using curve-fitting equations of connection model to the probable variables.
2. Economical design is possible to use reliability index and failure probability of applied load during structure plan, repair and reinforcement, because partial safety factor of connections can be expected as quantity.
3. The more moment-rotation curve of beam-column connection close to pin connection, the less value changes of reliability index or failure probability occur. This means ductile is increased if it closes to pin connections.
4. According to mutual relations of connectivity kinds, the close curves of moment-rotational angle make toward characteristics of pin connectivity, the variations of reliability index and probability of failure become the smaller.
5. When the same angle is used at parts of a single web angle and double angle, with respect to the same moment the reliability of the part of a double web-angle becomes nonlinearly larger than that of a single web-angle, however in terms of rotation displacement reliability index which corresponds to the same rotation displacement is equal regardless of kinds of connectivity parts.
6. Each parameter of a top and seat angle with a double web angle may be calculated by a simply linear composition of top-seat-angle and web-angle, but reliability index of them cannot be obtained by the linear relation and increases nonlinearly according to moments.
7. With probable analysis result of moment-rotation relationship in this study, the reliability indices of connections can be used as basic data for coding limit state design.

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