International Journal of Control and Automation
Vol.6, No.5 (2013), pp.159-170
http://dx.doi.org/10.14257/ijca.2013.6.5.15

Research Doubly-fed Induction Generator Dynamic Characteristics Based on Time-varying Parameters

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Abstract

According to the dynamic characteristics of doubly-fed induction generator (DFIG) which is changing with the wind speed, the dynamic mathematical models of DFIG wind turbines major subsystems' have been established, which includes the aerodynamic subsystem, drive train subsystem and generator subsystem. The whole nonlinear time-varying parameters dynamic mathematical model would be obtained to integrate the major subsystems' dynamic mathematical models, and the whole linear time-varying parameters dynamic mathematical model can be obtained to linearize the whole linear time-varying parameters dynamic mathematical model. To take the 2MW DFIG wind turbine as example, the proposed model is validated by the study on the dynamic characteristics of DFIG with wind speed variation, and the results show that the proposed model could provide a theoretical support for optimal control of the DFIG wind turbines.

Keywords: *DFIG wind turbines; time-varying parameters; dynamic characteristics; mathematical model*

1. Introduction

With the advancement of science and technology as well as the growth of industrial production, people's demand for energy is also increasing. However, the reserves of conventional energy such as coal, oil and natural gas and other non-renewable energy has been declining, so it is urgent to find new renewable energy to replace the non-renewable energy sources. As a new promising renewable source, wind energy has the advantage of clean, large storage capacity and easy development, and has been widely explored and utilized. Among various energy shares, the proportion of wind power generation has been increasing. But wind power generation also has shortcomings. The most significant one is the wind speed's variability and uncontrollability. The output power of wind turbines has cubic relationship with the wind speed. The fluctuations of wind speed bring about great difficulties to the control of wind turbine and the stable operation of the power system. In order to optimize the wind turbine control and to maintain the stability of the power system operation, it is significant to study the wind turbines' dynamic characteristics with the wind speed variation.

At present, domestic and foreign experts have been studying the dynamic performance of wind turbine. A simplified 3-order dynamic model of DFIG was established which takes the double component of rotor excitation voltage and the input mechanical torque as control variables, and the response characteristics of each physical variables between simplified 3-order dynamic model and precise model were compared

ISSN: 2005-4297 IJCA

[1-2]. The dynamic mathematical model of DFIG in the d-q coordinate system was establish in [3], and the resistance effect was studied in the case that wind turbine is operating in variable wind speed. Simplified dynamic model of wind turbine was established and the wind turbines' dynamic process and response process under the fault of power system are researched [4-5]. In order to research the dynamic response process of wind turbine under wind speed mutating, Large-scale dynamic model of wind turbine was established [6-12]. The dynamic characteristics of tower in wind turbine operating process was researched, and gave the tower's dynamic characteristics under the time-varying load was given [13-16]. The above scholars have studied the dynamic characteristics of wind turbine from different angles. According to their own research aims to establish the corresponding model of wind turbine, and the simulation analysis were carried out. But the time-varying parameters of the whole state equation of wind turbine have not been considered in the above researches, as well as the dynamic responses process of each component with the wind speed variation during wind turbines in the operation.

To solve the above problems, the dynamic mathematical models of major subsystems, including the aerodynamic subsystem, drive train subsystem and generator subsystem, have been established. The whole nonlinear time-varying parameters dynamic mathematical model of wind turbine was obtained to integrate these mathematical models, and the whole DFIG wind turbines' linear time-varying parameters dynamic mathematical model was obtained to linearize the whole nonlinear time-varying parameters dynamic mathematical model. To take the 2MW DFIG wind turbine as example, the proposed model is validated by the study on the dynamic characteristics of DFIG with wind speed variation.

$$1 \quad 2$$

$$WT \quad p$$

2. Mathematical Models of the Subsystems

2.1. Aerodynamic mathematical model

(1)

Wind turbine converts the wind energy into mechanical energy and its amount has the cubic relationship with the wind speed. The relationship can be represented by equation (1).

2

In Eq. (1), $_{PWT}$ is the mechanical power of the wind turbine, p denotes the air density, R represents the radius of wind wheel, is the weed speed, *fi* is the pitch angle, *2* is the blade top speed ratio, C_p is the wind energy utilization efficiency of wind turbine, the relationship among C_p, *fi* and *2* can be expressed as equation (2).

$$C\,/3{,}2 = 0.22(\quad 116 \quad -0.4/3 - \quad {}_{p} \quad , 2_i \quad 5)e \quad 2 \quad i \quad 3 \tag{2}$$

Figure 1 describes the relationship among C_p, *fi* and *2*. dynamic characteristics of DFIG wind turbines has been mainly studied in this paper considering the wind speed variation. In order to facilitate the analysis of the main problems, it is assumed that the doubly-fed wind turbine is running under the rated wind speed in this case. Under this condition, the pitch angle is 0□, and then C_p is only relevant to *2*.

2.2. Drive train mathematical model

Drive chain is a set of devices which connect the rotor shaft and the high-speed shaft of generator. Therefore, the rotor torque r_{WT} and generator electromagnetic torque r_G are taken as model inputs, and the speed of high-speed shaft Q_h is the output. Assume that the influence of the drive chain's structure characteristics (such as vibration, gear type, backlash, etc.) on its performance can be neglected; therefore, the drive chain has constant mechanical transmission efficiency within the whole speed range.

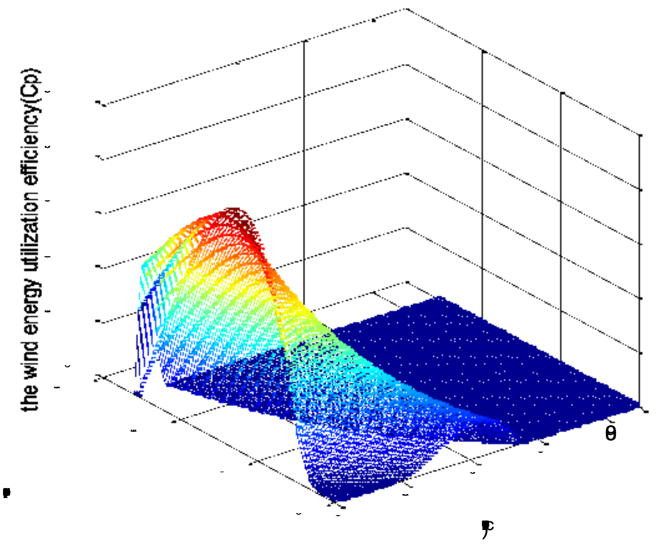

Figure 1. The calculating relations of C_p, β and λ

According to the different research purpose, wind turbine generators' drive chain is usually divided into rigid drive chain and flexible drive chain. Rigid drive chain is that the whole wind turbine generators' drive chain can be expressed as a single rigid coupling accelerator, and in the flexible drive chain the elastic deformation of drive shaft would be considered. Flexible drive chain, compared with rigid drive chain, describes its energy transfer process of the wind turbine generators' drive chain much more objectively and specifically, but its differential equations are complex. The structure and differential equation of rigid drive chain is relatively simple so as to facilitate the establishment of wind turbines' overall dynamic mathematical model. Thus, in this paper the rigid drive chain has been chosen for the mathematical modeling.

The structure of rigid drive chain is shown in Figure 2. In that, J_{WT} is the rotational inertia of rotor, J_l is the rotational inertia of low speed shaft and r_{WT} is wind rotor torque, i and η are the transmission ratio of gear box and the transmission efficiency, J_h is the rotational inertia

$$J_h \frac{dQ_h}{dt} = \frac{\eta}{i} P_{WT1}(Q, v) - P_{Gh}(Q, c)$$

of high speed shaft, *Error! **Reference source not found.*** is the electromagnetic torque, J_g is the rotational inertia of generator rotor.

Due to the role of accelerator, the electromagnetic torque Error*! Reference source not found.* is *i* times less than the rotor torque r_{WT}, while the angular velocity Q_h has increased *i* times, namely *Error!* ***Reference source not found.***=*i*Error! ***Reference source not found.***. Rigid drive chain model can be represented by equations (3) and (4).

(3)

(4)

In equations (3) and (4), T_{WT}(*Error!* ***Reference source not found.***,*v*) selects the wind speed *v* and low speed shaft's rotational speed *Error!* ***Reference source not found.***as parameters. T_G(*Error!* ***Reference source not found.***,*c*) selects *Error!* ***Reference source not found.*** and *c* as parameters, *c* stands for load variable, and it is a constant in the stable operation system. The calculation of J_h and J_l can be expressed by equations (5) and (6).

(5)

(6)

In (5) and (6), Error! **Reference source not found.**and*Error!* ***Reference source not found.*** are the inertia of overdrive gear.

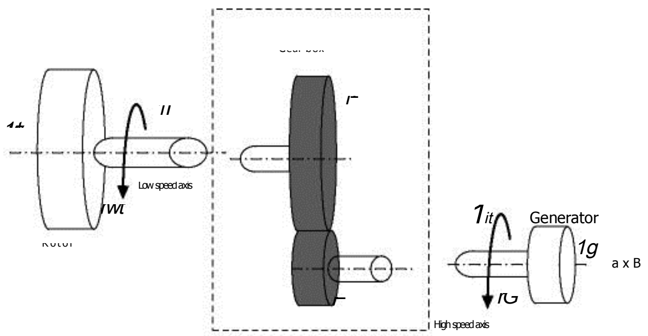

Figure 2. The structure of rigid drive train

2.3. Generator mathematical model

The electromagnetic torque equation of doubly-fed induction generator in (*d*, *q*) coordinate can be expressed by equation (7).

$$T_G = \left(\frac{3}{2}\right) pL_m (i_{Sq} i_{Rd} - i_{Rq} i_{Sd}) \tag{7}$$

In (7), *p* is the number of pole pairs, L_m is the mutual inductance between the stator and rotor, i_{Sd} , i_{Sq} ,i_{Rd} , i_{Rq} are the current component of the stator and rotor in (*d*, *q*) coordinate, the dynamic equations of current component are shown in (8).

In (8), *co* is the rotating frequency of rotor flux (rad/s), *co*=*pError*! ***Reference source not found.***, *Error!* ***Reference source not found.*** is the rotational angular velocity of generator's rotor, *cos* is the rotating frequency of stator flux (rad/s), *cos* =dO_S/dt, O_S is the rotating angle of stator flux, R_S and R_R are the resistance of stator and rotor, L_S and L_R are the inductance of stator and rotor, V_{Sd}, V_{Sq}, V_{Rd}, V_{Rq} are the voltage components of stator and rotor in (*d*, *q*) coordinate. The current and voltage in the equation (8) can be expressed in vector form as equation (9).

(8)

(9)

(10)

$[i_{Sd} \; i_{Sq} \; i_{Rd} \; i_{Rq}]$

$[V_{Sd} \; V_{Sq} \; V_{Rd} \; V_{Rq}]$

To take x as state variable and u as input variable, the state equations of doubly-fed induction generator can be expressed as 4-order linear state equations in equation (10).

$x = A(f1h)x + Bu$

To make $\sigma = 1 - L_m^2 / (L_S \cdot L_R)$, A and B can be expressed by the equations (11) and (12).

(11)

(12)

In Eq. (11), the coefficient matrix A is related to the angle velocity of high speed shaft ***Error! Reference source not found.***, and *Error!* ***Reference source not found.*** is related to the wind speed v. Therefore, the coefficient matrix A is a time-varying parameter matrix, which will change with the wind speed.

3. Overall Mathematical Model of the DFIG System

As mentioned before, the aerodynamic mathematical model, the drive chain mathematical model and the generator mathematical model of doubly-fed induction generator have been proposed. How to integrate the above three separate sub-models into a whole dynamic mathematical model of the doubly-fed wind turbine will be studied as follow.

The calculation equation of wind wheel torque T_{WT}(*Error!* ***Reference source not found.***,*v*) is shown as equation (13).

(13)

Where *CT*= C_p */2, 2=R·Error!* ***Reference source not found.****/v*.

Equation (3) can be converted to equation (14).

(14)

T_{WT}(*Error!* ***Reference source not found.***,*v*) and T_{WT}(*Error!* ***Reference source not found.***,*v*) in (14) can be replaced by (13) and (7) as shown in equation (15).

□*d*f2*h* - _ *i* 3)*pL* (*i* *Rq* *i*)

*v*0.5 □□ 21*e*Cr(*2*) *dth* *Jh* (15)

Combinations of the equations (15) and (8) can be obtained as a whole state equation of the doubly-fed wind turbine which expresses as the equation (16).

diRd □*V*Rd *R*R *L*m *di*Sd □(□ □)□*i* □*Lm i*

□ *i* Sq □ *L dt*

□*L*R

di V R i L di

Rq Rq R *m Sq* *i Lm*

J

(16)

As shown in eq. (16), the first expression at the right of the equal sign ... 0.5 ... the fifth differential equation ... [illegible] ... nonlinear ... In order to take advantage of the knowledge of the linear state space to carry out research, the whole model needs to be linearized.

4. System Mathematical Model Linearization

Since the nonlinear characteristic of the state equations of the wind turbine, it is not conducive to the study of dynamic characteristics; and the state equation is linearized in this section as follow.

For the grid-connected doubly-fed wind turbine, system input variables should include not only the wind speed *v*, but also include the stator and rotor voltage; while the state variables of the system contains the stator current (i_{Sd}, i_{Sq}) , rotor current (i_{Rd}, i_{Rq}), and also the high speed shaft angular velocity *Error!* ***Reference source not found.***. Therefore, the overall system state variables and input variables are given in the eq. (17).

$$x = [x_1\ x_2\ x_3\ x_4\ x_5] = [i_{Sd}\ i_{Sq}\ i_{Rd}\ i_{Rq}\ \ _h]$$

$$u = \qquad (17)$$

From the characteristic of wind turbine, it is known that the rotor torque Γ_{WT}(*Error!* ***Reference source not found.***, *v*) and low speed shaft angular velocity *Error!* ***Reference source not found.*** were relative to the wind speed *v* which is as shown in Figure 3. In this diagram, electromagnetic torque operation line is located on the right side of the wind wheel torque curve. When the wind turbine operates at the stable operating point OP, as shown in the system dynamic equation (16), the current state of the generator can be described by six dimensional vector (i_{Sd}, i_{Sq}, i_{Rd}, i_{Rq})=(,).

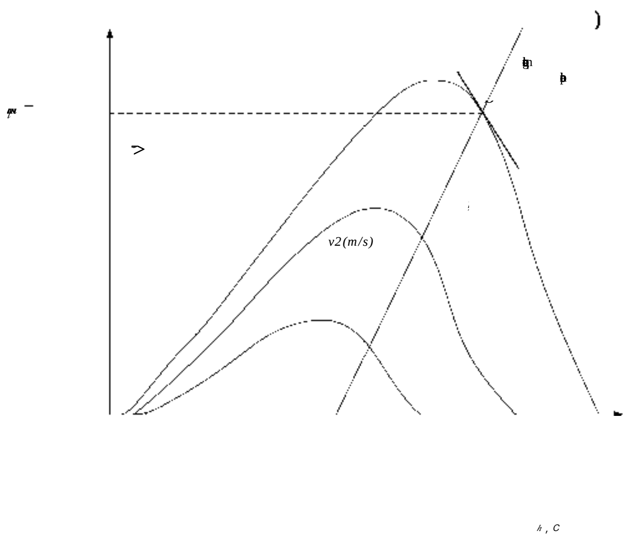

Figure 3. The calculating relations of *Error! Reference source not found.*,Error*!* *Reference source not found.* and Error*! Reference source not found.

$$f_A\ x \;=\; A\ i \quad A\ i \qquad (18)$$

121(rad/s)

ft sd

A_i A_i A_Q □ v_1 (m/s) When the running state of the doubly-fed wind turbine has very small changes near the stable operation point OP, accordingly the state variables and input variables would also have small changes, the value of this change as shown in equation (18).

□ $s d$

q R R h□
q K R R

Based on the theory of infinitesimal calculus, a tiny variation of the system could be calculated by equation (19).

(19)

(20)

In (19), $\bigtriangledown y$ is the change of the system output variable, $\alpha f/\alpha z$ is the derivative of the system equation, and $\bigtriangledown z$ is the changes of the system input variables.

When the wind turbine is running at stable operation point OP, the derivation of the first equation in (10) could be obtained according to the derivative rules of the equation (19) which could be described as (20) and (21).

(21)

1

□ □ □ *f* *B*

Lm

□*L* *S*

0 *L* *S* □ *R* *L*

The derivative values of Γ_{WT}(*Error!* ***Reference source not found.***,*v*) in eq. (13) at the steady-state operation running point OP could be obtained by taking the derivation of *Error!* ***Reference source not found.*** and *v* respectively , as shown in (22).

2

γπpR3

(22)

1 2

In (22), γ = γ(A)=C (A). AlC$_p$(A)-1.

The equation (3) can be converted to the equation (23).

0

From rigid drive chain it is known that , and equation (23) can be changed to equation (24).

(24)

So far, the derivative values operating at the point OP of all the equations in equation (16) have been obtained. The system dynamic changes at point OP can be represented by equation (25).

(25)

In (25), $x=[Ai_n\ Ai_n\ 4,\ A_i An_h]$, , the expression of and could be shown as the equation (26) and (27) respectively.

(26)

frequency 50 Hz, the radius of the wind wheel $R = 45m$.

(27)

$\square$ L L

s R

0

0

0

$\square$ L $_R$

0

^{2} $\square$ $\square$ $\square\square\square$ R_3 ^{cr} $(\square)$

i J 2

5. Simulation Analyses

To take a doubly-fed wind turbine as example, the parameters of this wind turbine for stator resistance $R_s = 0.00551$, stator inductance $\mathbf{L_s = 0.0004H}$, rotor resistance $Rr = 0.008951$, the rotor inductance $Lr = 0.00031$H, the mutual inductance of stator and rotor , and

the number of pole pairs p 2 , the outlet voltage of generator , system

Figure 4 shows the eigenvalue changes of the state matrix of double-fed wind turbine when the wind speed varies from 4 to 14m/s. The direction of the arrow represents the direction of the wind speed increasing, and the polar points in the ellipse represent the coordinate position of the rotor poles. As shown in this figure, the eigenvalue of driving chain and the polar coordinates of the stator eigenvalue changes along with the wind speed changes accordingly, while the polar coordinates of the rotor eigenvalue change a little.

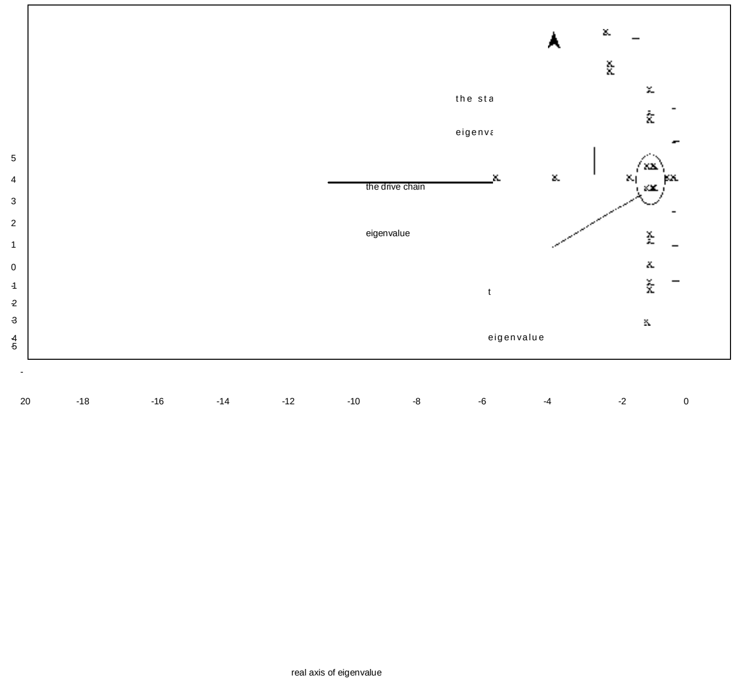

Figure 4. State matrix's eigenvalue of system

Figure 5 is the change process of the eigenvalue of the doubly-fed wind turbine drive chain along with the wind speed variation. As shown in figure 5, the coordinate values of the drive chain eigenvalue are negative real numbers without any of the imaginary axis part. It corresponds to the choice of first-order rigid drive chain model in this case, and this eigenvalue is decreasing with the increasing of wind speed. In this diagram, the direction of the arrow represents the direction of the wind speed increasing. When the wind speed is ranging among 4 to 14m/s, the eigenvalue of the drive chain is reducing. That is to say the dynamic response of the system is continuously growing, and the dynamic response time is being shortened.

From Figure 4, the imaginary part of the rotor's eigenvalue has no changes, only the real part has very small changes. Figure 6 shows the real part of the eigenvalue of the doubly-fed wind turbine rotor. From the changing process of the rotor eigenvalue's real part, it is known that in the process of the wind speed increasing, the real part of rotor eigenvalue is decreasing slowly, and the dynamic response process of rotor is increasing gradually, the dynamic response time is being shortened gradually. It shows that there is no change in the dynamic oscillation characteristics of rotor because the imaginary part of rotor has no change.

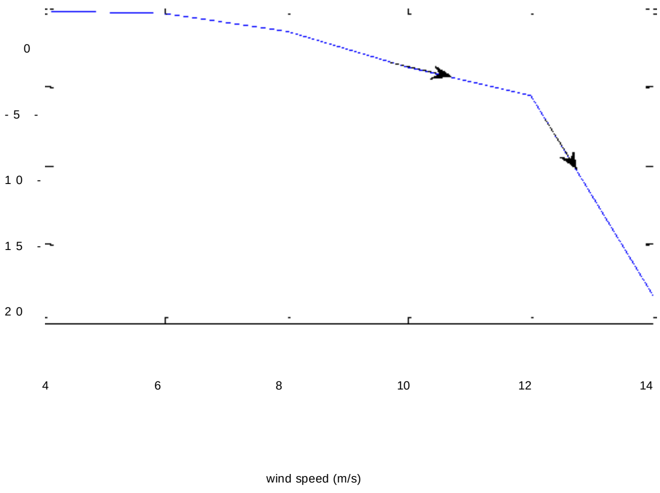

Figure 5. Drive train's eigenvalue

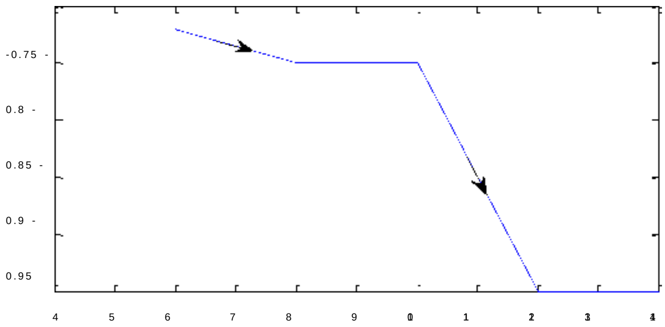

Figure 6. Rotor eigenvalue' real part

Figure 7 describes the imaginary part of the stator eigenvalue of doubly-fed wind turbine ... real part of stator does not change, indicating that there is no changing of the stator dynamic response.

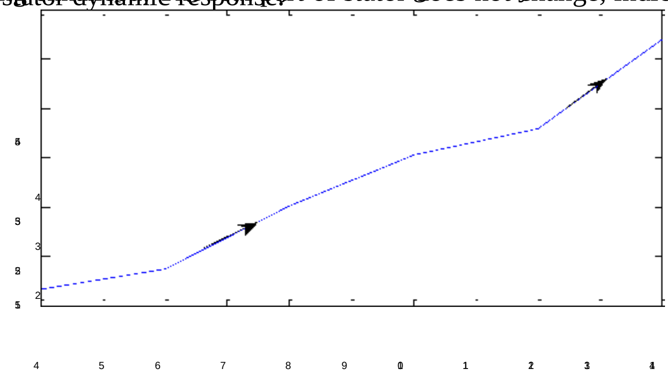

Figure 7. Rotor eigenvalue' imaginary part

In this section, the dynamic characteristics of the doubly-fed wind turbine operating during the whole wind speed variation process were studied. Moreover, it is proved that the characteristics of the drive chain, the generator stator and rotor would change along with the wind speed changes, and the law of this dynamic characteristic has been presented. It provides the theoretical support for the optimization control of the doubly-fed wind turbine.

6. Conclusion

In this paper, an overall nonlinear dynamic mathematical model of doubly-fed wind turbine with time-varying parameters has been established based on the energy conversion principle of doubly-fed wind turbine and linearized to facilitate its dynamic characteristics studying. To take the 2WM doubly-fed wind turbine as example, the proposed model has been validated on the condition that the wind turbine is operating when the wind speed varies from 4 to 14m/s, and the results show that:

1) The dynamic response of the doubly-fed wind turbine drive chain is growing with the increasing of the wind speed, while the dynamic response time is decreasing accordingly;

2) Doubly-fed wind turbine rotor's dynamic response process would strengthen gradually, and the dynamic response time would shorten, but the dynamic oscillation characteristics of the rotor would not change;

3) The dynamic response of the stator of the doubly-fed wind turbine would not change, but its dynamic oscillation characteristics would gradually increase.

Acknowledgements

The work was supported by the national Ministry of science and technology 863 major project of china (SQ2010AA0523193001.

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