the branches of $A_{eq}(\alpha)$ corresponding to α with different signs should be symmetrical. To describe the observed phase dependence $A_{eq}(\alpha)$ of the lunar areas, different model functions may be used. The simplest phase function, which is very appropriate for the phase-angle range 10–40°, is

$$A_{eq}(\alpha) = A_0 \exp(-\mu\alpha) \tag{7}$$

where μ characterizes the slope of the function.

Eq. (1) ignores the resolved lunar topography. To take this into consideration, one needs to use the following formula:

$$f(\alpha) = F_{obs}(\alpha,\beta',\gamma')/D(\alpha,\beta',\gamma'), \tag{8}$$

where $\gamma' = \gamma + \Delta\gamma$ and $\beta' = \beta + \Delta\beta$ are the photometric coordinates accounting for the topography. The values $\Delta\gamma$ and $\Delta\beta$ can be expressed through the local slopes of the resolved lunar relief.

Let us define the local slope $\vec{\mathbf{r}}$ as the declination of the local surface normal $\vec{\mathbf{n}}$ from the global one $\vec{\mathbf{N}}$, i.e., we consider $\vec{\mathbf{r}} = \vec{\mathbf{N}} - \vec{\mathbf{n}}$ where vectors $\vec{\mathbf{n}}$ and $\vec{\mathbf{N}}$ are normalized to unity. The local slopes of the surface affect the local illumination/observation geometry and, hence, the photometric coordinates γ and β of a given point. The slope disturbance of a small surface site (planar element) is equivalent to the displacement of the site to the point with new photometric coordinates γ' and β' and, therefore, new selenographic coordinates $l' = l + \Delta l$ and $b' = b + \Delta b$, where l and b are the selenographic longitude and latitude, respectively. This new point has a normal to the undisturbed surface parallel to the original local normal. We denote the longitudinal and latitudinal components of the local slope $\left|\vec{\mathbf{r}}\right|$ as r_l and r_b:

$$r_l = \left|\vec{\mathbf{r}}\right| \sin\psi, \quad r_b = \left|\vec{\mathbf{r}}\right| \cos\psi, \tag{9}$$

where ψ is the azimuth angle of the projection of the local normal on the average plane, which is measured from the direction to the north pole. The quantities r_l and r_b are related to Δl and Δb by the obvious equations

$$rl = \Delta l \cos b, \quad rb = \Delta b. \tag{10}$$

Formulas (9) and (10) are approximate (Korokhin and Akimov, 1997); they work well for the case of relatively small topography slopes (up to 10^{o}), when spherical triangles may be replaced by planar ones. Such conditions are well suited for lunar observations, since larger slopes on the ~3 km bases provided by Earth-based telescope observations of the Moon are rare.

In this work we develop a more sophisticated algorithm without such restrictions. The relations between the selenographic and photometric coordinates are defined by the expressions

$$\cos\alpha_0 = \sin b_{obs} \sin b_S + \cos b_{obs} \cos b_S \cos(l_S - l_{obs}), \tag{11}$$

$$\cos i = \sin b \sin b_S + \cos b \cos b_S \cos(l_S - l), \tag{12}$$

$$\cos e = \sin b \sin b_{obs} + \cos b \cos b_{obs} \cos(l_{obs} - l), \tag{13}$$

$$\cos\varphi = (\cos\alpha - \cos e \cos i)/\sin e \sin i, \tag{14}$$

where α_0 is the phase angle at the lunar disk center, and φ is the azimuthal difference between the incident and reflected rays at the point with selenographic coordinates (l, b). The values l_S and b_S are, respectively, selenographic longitude and latitude of the Sun, l_{Obs} and b_{Obs} are the same for the observer. The system (2) relates the angles i and e with γ and β.

To take into account the variations of the photometric conditions over the lunar disk, the following formulas are used

$$\gamma = \gamma_0 + \Delta\gamma, \quad \beta = \beta_0 + \Delta\beta, \quad \alpha = \alpha_0 + \Delta\alpha, \tag{15}$$

where γ_0 and β_0 are the photometric coordinates of the lunar disk center,

$$\Delta\alpha = \vartheta_L \cos\beta_0 \sin\gamma_0, \tag{16}$$

$$\Delta\gamma = \vartheta_L \sin\gamma_0 (1 + \cot\alpha_0 \cot\gamma_0 (\sin\beta_0)^2)/\cos\beta_0 \quad \text{at} \quad \gamma_0 \neq 0, \tag{17}$$

$$\Delta\gamma = \vartheta_L \cot\alpha_0 (\sin\beta_0)^2/\cos\beta_0 \quad \text{at} \quad \gamma_0 = 0, \tag{18}$$

$$\Delta\beta = \vartheta_L \sin\beta_0 \cos\gamma_0 (1\text{-}\cot\alpha_0 \tan\gamma_0), \tag{19}$$

where ϑ_L is the angular radius of the Moon seen from the Earth.

For calculation of selenographic coordinates of points on images of the Moon given in the perspective projection, some equations from (Shalygin et al., 2003) were used. The formulas and code are available on the web-site http://www.astron.kharkov.ua/dslpp/cartography/index.html.

This short introduction to terminology and definitions enables us to state the gist of our approach to retrieve information about the relief, albedo, and the parameters of phase-function slope from photometric images. We suppose that there is a rather large set of absolutely calibrated images obtained for the same scene at different phase angles. Hence, for each point of the lunar surface a phase dependence of the equigonal albedo can be plotted. We may numerically minimize the standard deviation of the observed $A_{eq}(\alpha)$ from the model function (7) varying simultaneously the parameters r_l, r_b, A_0, and μ. This automatically provides minimization of brightness variations caused by topographic slopes. For the minimization, we use the Nelder–Mead method (Nelder and Mead, 1965).

The described procedure is possible for the slope component oriented along the illumination direction. For Earth-based observations at large phase angles, the incident solar rays are nearly co-linear with the lines of selenographic latitude. Hence, the reconstructed relief profiles pass along latitude lines, and the coordinates of points on each profile are values of longitude. Therefore, we term this slope component longitudinal, r_l. Note, that in geography such a component is named "zonal".

Thus, we fit the parameters r_l, A_0, and μ, using a set of observations of the Moon carried out at various phase angles before and after full-moon. We map simultaneously the topographic slopes, albedo A_0, and the parameters of phase function compensating for the influence of the resolved topography.

3. Mapping the topography slopes and parameters of phase function

For the mapping we used data of absolute photometry of the Moon carried out in 2006 at the Maidanak Observatory (Uzbekistan) with a 15-cm refractor at red light 610 nm (Velikodsky et al., 2010). The data set of 12 maps of absolute albedo at α = 12.81°, 14.73°, 16.21°, 17.78°, 18.74°, and 21.02° before full-moon and at α = 12.33°, 13.42°, 14.19°, 15.07°, 16.19°, and 21.79° after full-moon were used in our analysis. The observation data were selected to have maximally symmetric values of phase angles before and after full-moon. The range of phase angles approximately from 12° to 22° allows us to use a relatively simple (one exponent) model phase function (7). The maximal phase angle 22° allows us to map almost the full visible disk in contrast to our previous work (Korokhin and Akimov, 1997). The resolution element of our data is approximately 3.2×3.2 km^2 in the lunar disk center. This is the minimal base of roughness.

Before calculations all images have been additionally co-registered using an algorithm of soft co-registration (Kaydash et al., 2009a) to compensate for the influence of Earth's atmospheric turbulence that deforms the lunar images. Fig. 1 presents a map of A_0, i.e., the albedo distribution that is not influenced by topography. We note that this is