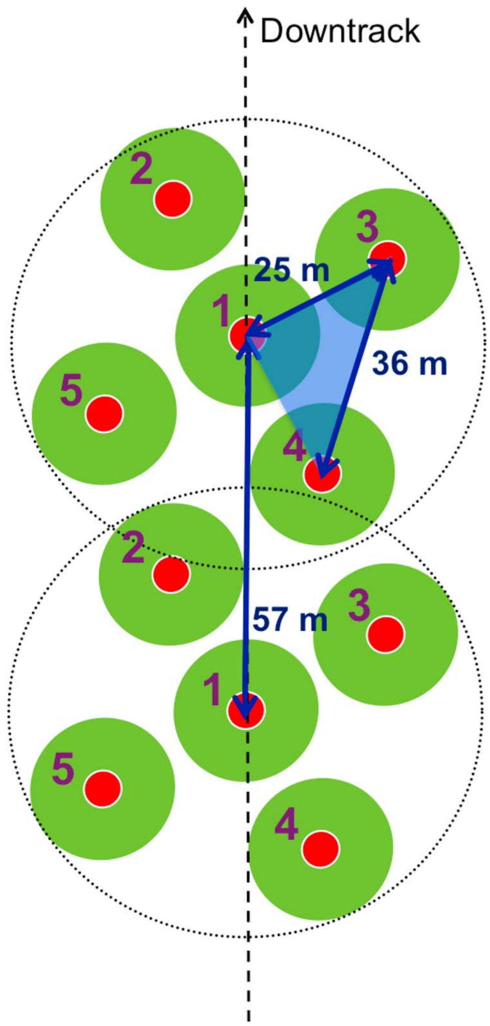

Figure 1. Plan view of two consecutive LOLA shots with spot numbers labeled. The shot-to-shot distance is ~57 m, and the smallest point-to-point baseline available is ~25 m. An example of a triangle used to calculate bidirectional slopes is shaded in blue. Red circles indicate the illuminated footprint of each laser spot, while green circles represent the field of view of each detector.

3180 tracks from the commissioning and mapping mission phases, acquired from 17 September 2009 to 9 March 2010, to compute and analyze a variety of parameters describing surface slopes and roughness. The data were processed to remove anomalous points (due to instrumental effects such as noise) and are spaced ~57 m apart along track and on average ~3.8 km across track at the equator and closer at the poles. Additional data have narrowed the cross-track spacing to ~1.8 km at the equator [*Smith et al.*, 2010b].

3. Global Surface Roughness of the Moon

[5] Quantitative measures of surface roughness have been defined in the literature in a number of ways. Here we investigate several measures of surface roughness, both in the interest of robustness in characterizing roughness units and to facilitate comparison with the literature. For one-dimensional slopes, we examine the RMS slope, the median absolute slope, and the median differential slope for a variety of horizontal scales, as well as the Hurst exponent, which describes how slopes scale with baseline (where the baseline is the horizontal length scale over which the slope is measured). In addition, LOLA's five-spot pattern allows for the calculation of two-dimensional slopes by fitting a plane to a set of three points along the track, resulting in the magnitude and direction of steepest descent.

3.1. RMS and Median Slopes

[6] The RMS slope is routinely calculated for the statistical analysis of topography because radar reflection scatter is often parameterized with this metric. In one dimension, it is defined as the RMS difference in height, Δz, between each pair of points (also known as the deviation, ν) divided by the distance between them, Δx:

$$s(\Delta x) = \frac{\nu(\Delta x)}{\Delta x} = \frac{1}{\Delta x}\left\langle [z(x_i) - z(x_{i-1})]^2 \right\rangle^{\frac{1}{2}}, \quad (1)$$

where the angle brackets indicate the mean. However, because the RMS slope depends on the square of the deviation, this parameter is quite sensitive to outliers; this poses a significant problem because the slope-frequency distribution for natural surfaces is often non-Gaussian with strong tails. The median absolute slope is a more robust measure of typical slopes, as it is less affected by long tails in the distribution.

[7] To find the RMS and median slope in the along-track direction, point-to-point slopes were calculated for each track, stored at the midpoint, and averaged according to (1) within 0.5° (~15 km) sliding windows, each spaced 0.25° (~7.5 km) apart. The LOLA lasers have a firing frequency of 28 Hz, corresponding to a shot density of approximately 540 shots per degree down track, or roughly 270 shots per window at best. However, owing to noise and instrument performance issues, missing points are not uncommon. Since the RMS slope is sensitive to the number of points, N, included in each window, uneven N across the surface can introduce variations in the RMS slope map that are not due to real roughness features. To minimize this bias, windows were only considered valid if more than 250 measurements contributed to the average in that location. The median absolute (unsigned) slope is far less sensitive to the number of points in each bin. Given LOLA's ground spot pattern, the smallest baseline available for slope calculations is about 25 m, the distance on the surface from the center spot to any of the four corners (Figure 1).

[8] One-dimensional slopes calculated along profile underestimate the true gradient of the surface wherever the direction of steepest descent diverges from the along-track direction. At the smallest scales, this ambiguity can be resolved by computing the slopes in two dimensions from multiple points within each laser shot. We use vector geometry to compute the plane passing through three spots, recording the magnitude and azimuth of the slope. One such triangle appears as a shaded region in Figure 1. The effective baseline of the slope is taken to be the square root of the area of the triangle. The slope values are then binned as