environment of Phobos means that the direction of the vector defining the apparent acceleration due to gravity is not simply related directly to the local topography of the irregular body, but is modified by the rapid rotation and the tidal forces due to the proximity of Mars. Thomas (1993) gave values for the variation over the surface of Phobos of the dynamic height, defined as the height that corresponds to the difference in gravitational potential between any geographic point and a reference point chosen as the south pole. The difference between the dynamic heights of any two geographic locations, multiplied by the local value of the apparent acceleration due to gravity, defines the energy per unit mass gained or lost in traveling between the locations, and so the inverse tangent of the difference in dynamic height divided by the separation distance along the surface produces an angle, α, that is, in the Phobos environment, the topographic surface slope as it would normally be defined.

Thomas (1993) tabulated dynamic heights, and the total effective acceleration due to gravity, at 2° intervals of latitude and longitude across Phobos, and we use this spacing to trace boulder motion. For the purpose of defining distances along the equator we treat Phobos as an ellipsoid, symmetrical about its long axis, with semi-major axis $a=13.22$ km and semi-minor axes $b=9.97$ km (see also Wählisch et al., 2010; Willner et al., 2011). The x, y, and z Cartesian co-ordinate axes are defined such that west longitude is 0° at ($x=+a$, $y=z=0$), 90°W lies at ($x=z=0$, $y=-b$), and the north pole is at ($x=y=0$, $z=+b$) (see Fig. 6). All sections in the equatorial plane are ellipses described by

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1. \tag{10}$$

If θ is the polar angle at the center of symmetry of the ellipse, measured positive eastward from the x axis between longitude zero and any point on the ellipse at radial distance r from the center of symmetry, Fig. 6(b) shows that

$$x=r\cos\theta \quad \text{and} \tag{11}$$

$$y=r\sin\theta. \tag{12}$$

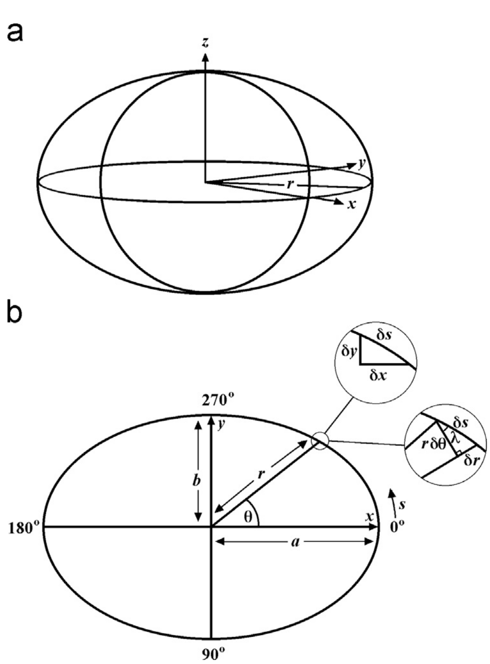

Fig. 6. (a) Definition of the Cartesian co-ordinate system for an ellipsoid model of Phobos; (b) relationship of Cartesian system to longitudes, semi-major (a) and semi-minor (b) axes, surface distance, s, and (r, θ) co-ordinates. The positive x-axis points toward Mars and the negative y-axis is the direction of Phobos' orbital motion.

These relationships allow r to be expressed as a function of θ given a and b by

$$r^2=\frac{1}{\left((\sin^2\theta/b^2)+(\cos^2\theta/a^2)\right)}, \tag{13}$$

and so any increment δr in r due to an increment $\delta\theta$ in θ is found, by differentiating Eq. (13), to be

$$\delta r=\frac{\sin\theta\cos\theta\ \left((1/a^2)-(1/b^2)\right)}{\left((\sin^2\theta/b^2)+(\cos^2\theta/a^2)\right)^{3/2}}\delta\theta. \tag{14}$$

We need to relate any change in polar angle ($\delta\theta$) to the corresponding change that it causes in the arc length measured along the curve of the ellipse, s (i.e., δs). The upper inset in Fig. 6b shows that

$$\delta s^2=\delta x^2+\delta y^2, \tag{15}$$

and by differentiating Eqs. (11) and (12) and substituting for δx and δy we find that

$$\delta s^2=\delta r^2+r^2\,\delta\theta^2. \tag{16}$$

Thus Eqs. (14) and (16) allow δr and hence δs to be related to $\delta\theta$, which we take as the 2° interval of latitude and longitude used by Thomas (1993).

We can now equate the total force acting on a boulder, made up of the positive apparent gravitational force acting down the slope α derived from the dynamic height change, and the negative drag force due to the resistance of the regolith, given by the right-hand side of Eq. (8), to the boulder mass multiplied by its acceleration. Again using the substitution $du/dt=u(du/ds)$, we find that

$$\left(\frac{4}{3}\right)\pi r^3\rho_b u\frac{du}{ds}=\left(\frac{4}{3}\right)\pi r^3\rho_b g\sin\alpha-Q\rho_r g r^3\left[S+T\left(\frac{u^2}{g r}\right)\right], \tag{17}$$

which simplifies to

$$u\frac{du}{ds}=g\sin a-\left(\frac{3}{4\pi}\right)Q\left(\frac{\rho_r}{\rho_b}\right)g\left[S+T\left(\frac{u^2}{g r}\right)\right]. \tag{18}$$

Given an assumed initial boulder speed u_i, Eq. (18) can be integrated numerically to provide the variation of u with distance s along the surface from the starting point. The parameter values used are a regolith density, ρ_r, of 1000 kg m^{-3}, a boulder density, ρ_b, of 1800 kg m^{-3}, a boulder radius of 250 m, and two values of a. For the first value, $a=0.1$, then $q=0.07692$, $f=0.38462$, $Q=0.00994$, $S=3.9934$, and $T=16.38$; where $a=0.125$, then $q=0.11765$, $f=0.47059$, $Q=0.02845$, $S=4.0166$, and $T=10.71$. To choose a suitable initial boulder speed we combined the values of escape velocity as a function of geographic position and launch direction given by Dobrovolskis and Burns (1980) and Davis et al. (1981). Their values were generally calculated for Phobos at its present orbital distance from Mars, but they can be adjusted for greater distances at earlier times (Thomas (1998) suggested that ~1.2 times the present orbital distance is consistent with the distribution of ejecta around Stickney). Fig. 7 shows the resulting variation of escape speed with west longitude for an object traveling horizontally to the east in the region east of Stickney.

Fig. 8 shows examples of calculated tracks for 250 m-radius boulders ejected horizontally along the equator from a starting point at the approximate location of Stickney's east rim, longitude 30°W, at an initial speed of 5.0 m s^{-1}. This speed is less than the local escape velocity of ~5.4 m s^{-1} shown in Fig. 7. The distances are measured along the curved surface of Phobos. The shortest eastward distance, ~10.3 km, corresponds to $a=0.125$, for which the friction losses due to the depth of penetration of the boulder into the regolith are the greatest. Decreasing a to 0.1, the minimum penetration suggested by the groove shape measurements,