saturated) wetting front (<10% GWC) that reaches a maximum depth of ~15 cm; visual inspection revealed that most of this moisture is quickly lost via evaporation within hours after infiltration. In most cases, snowfall at the ground surface sublimes before melting (e.g., Fountain et al., 2010).

5. Numerical modeling

5.1. Assumptions and model format

Vapor fluxes through cold and dry tills are governed primarily by two mechanisms: molecular diffusion of vapor in pore spaces and advection of air through the material. Other forces influence vapor transport, including osmotic pressure and nonisothermal vapor flux; but it has been reported that Fickian diffusion processes dominate transport in extreme cold environments (Chevrier et al., 2007, 2008; Hudson et al., 2007). Our vapor-diffusion model ignores the very minor effects of Knudsen diffusion and does not specifically track vapor density beyond saturation (as may occur with the formation of hoar frost). Results, however, can be used to infer times when secondary ice would likely form in pore spaces within the Mullins till. Such secondary ice would decrease till porosity, retard vapor diffusion, and cause our model to overestimate ice loss; hence our modeled values for ice loss presented below are most appropriately viewed as maxima, not minima. Lastly, we do not consider the effects of adsorption/desorption of water vapor on soil particles in this treatment. Although such processes likely play a role in modulating vapor diffusion, recent studies suggest that the effects are typically short lived and most probably of second-order importance; further research is required (Schorghofer and Aharonson, 2005).

The molecular diffusion of water vapor in pores in Mullins till is expressed using Fick's Second Law,

$$\frac{\partial \rho}{\partial t} = D\frac{\partial^2 \rho}{\partial z^2} \tag{1}$$

where ρ represents water vapor density, t the time, D the diffusivity of water vapor ($0.16\ \mathrm{cm^2\ s^{-1}}$) (Mellon and Jakosky, 1993; McKay et al., 1998), and z is equal to distance. To address vapor diffusion through a porous medium, we then introduce variables to express porosity, ϕ, and tortuosity, b (Eq. 2), into Eq. (1). Vapor diffusion flux can now be expressed as

$$\frac{\partial \rho}{\partial t} = \left(\frac{\phi D}{b}\right)\cdot\frac{\partial^2 \rho}{\partial z^2} \tag{2}$$

The vapor density for boundary and initial conditions is calculated from the water vapor pressure (e), which can be derived using the Clausius–Clapeyron equation

$$e = e_0 \cdot \exp\left[\frac{L}{\Re_v}\cdot\left(\frac{1}{T_0}-\frac{1}{T}\right)\right]\cdot\frac{RH}{100} \tag{3}$$

where e_o and T_o are constant parameters equal to 0.611 kPa and 273 K, respectively; $L/\Re_v$ is equal to the latent heat of deposition divided by the gas constant of water vapor which equals 6139 K; and T and RH are the HOBO™ Smart Sensor measured temperature and the relative humidity, respectively. The relationship between vapor pressure and vapor density can be calculated using the ideal gas law as follows:

$$e = \rho \Re_v T \tag{4}$$

where the equation is solved for the vapor density. We solve Eq. (2) with the implicit finite differences method to calculate vapor fluxes through the till using 1 cm grid spacing. Ice loss is calculated from the net vapor flux at the ice–till interface.

We assume that the temperature of air in pores within Mullins till is the same as that recorded for surrounding sediment. In the rare cases where soil temperatures were unavailable at depth, we calculated the expected soil temperatures at depth by propagating measured soil surface temperatures using the following one-dimensional heat diffusion equation solved using finite differences:

$$\frac{\partial T}{\partial t} = \kappa\frac{\partial^2 T}{\partial z^2} \tag{5}$$

where T represents temperature, t is time, κ is the thermal diffusivity, and z is depth. Thermal diffusivity for the various facies of Mullins till was calculated from our measured data as

$$\kappa = \frac{\pi}{P}\left(\frac{z_2 - z_1}{\ln(\delta_1/\delta_2)}\right)^2 \tag{6}$$

where P is the period, and δ is half-amplitude of the temperature variation at depth z. Calculated thermal diffusivities are $2.3\times10^{-7}\ \mathrm{m^2\ s^{-1}}$ for the weathered facies; $8.8\times10^{-7}\ \mathrm{m^2\ s^{-1}}$ for the fresh facies; and $5.3\times10^{-7}\ \mathrm{m^2\ s^{-1}}$ for both the sand-wedge facies (same values for active and relict sand wedges). These values fall within the range of thermal diffusivities calculated for various sediments throughout the Dry Valleys region (Campbell et al., 1997; Campbell and Claridge, 2006). Boundary temperature conditions for the ice surface were established from ice core temperatures at 5 m depth.

The boundary conditions for relative humidity (RH) are set to the measured value for atmospheric temperature at the till surface and are assumed to be 100% just above the buried ice surface. Initial conditions include a linear function of RH between these two boundary conditions. The model then solves Eq. (2) to determine vapor density as a function of depth and computes cumulative vapor flux.

5.2. Modeling exercise 1: elucidating the role of till facies on ice sublimation

5.2.1. Strategy

To examine the extent to which changes in the porosity and thermal diffusivity of facies within Mullins till alter sublimation of underlying ice, we modeled ice losses beneath three simulated till sections: one in which the till was composed entirely of the fresh facies; a second in which it was composed entirely of the weathered facies; and a third in which it was composed entirely of the sand-wedge facies. For all model runs, till thickness was arbitrarily set at 50 cm, and meteorological forcing came from data collected at the Mullins terminus (site 5) in 2006 (Tables 1a, 1b, and 1c). Subsurface temperature profiles were created for each model run according to thermal diffusivities calculated in Eq. (6) above.

5.2.2. Results

The results show that variations in till texture and thermal diffusivity impact vapor diffusion and loss of buried ice. Annual losses beneath the fresh, weathered, and sand-wedge facies are 0.0603, 0.0601, and 0.0636 mm a^{-1}, respectively. Setting sublimation losses beneath the fresh facies as a base level value and assigning percentages (higher or lower than base level) for the remaining two facies indicates that ice loss is greater beneath the sand-wedge facies by 5.47% and lower beneath the weathered facies by 0.33%. Although these net deviations are relatively small, they help explain some of the meter-scale topographic relief observed at the surface of buried glacier ice in this region (e.g., Linkletter et al., 1973; Bockheim, 2002; Marchant et al., 2002; Sletten et al., 2003). For example, over timescales of 10^5 years, the increased sublimation beneath active sand wedges could result in the development of marginal polygon