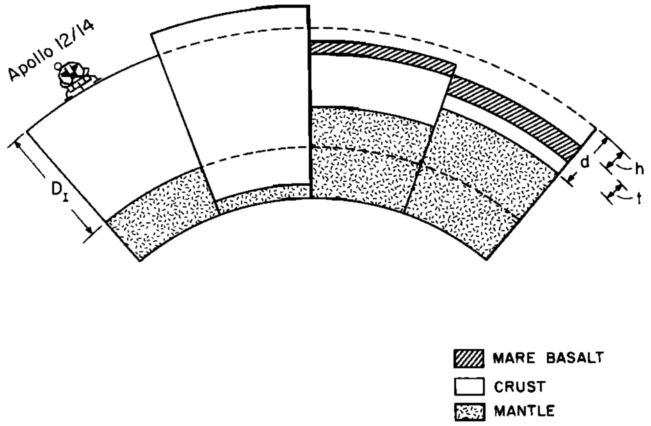

Fig. 3. Schematic cross section of the crust and upper mantle, illustrating how the assumption of premare isostasy is used to constrain the distribution of anomalous mass in lunar mare regions. We use equation (6) and the observed topography h to constrain the thickness t of mare basalt and the depth d to the Moho in mare blocks. The seismically determined crustal thickness in the vicinity of the Apollo 12 and 14 landing sites serves as a reference section.

regions of ancient premare structure and thin mare fill in the Oceanus Procellarum–Mare Cognitum area, it is reasonable to assume that the degree of isostasy in the region is nearly complete. We therefore adopt a 55-km-thick layer of nonmare crustal material of density ρ_c as the standard section for (6). With this assumption the model for the nearside crustal structure developed in this study automatically matches the crustal thickness in the vicinity of the Apollo 12 and 14 landing sites.

Inversion for Crustal Structure

To determine the Moho configuration and thickness of mare basalt on the lunar nearside, we invert the Bouguer anomaly data, subject to the adopted assumptions and constraints. The inversion scheme iteratively improves on an initial estimate of the crustal structure. The criterion used to determine a best fit model is the minimization of the root-mean-square residual gravity anomaly:

$$\text{rms residual} = \left[\sum_{k=1}^{N} (g_k^{\text{obs}} - g_k^{\text{calc}})^2 / N \right]^{1/2} \tag{7}$$

where N is the number of gravity anomaly observations, g_k^{obs} is the kth observed gravity anomaly, and g_k^{calc} is the kth gravity anomaly calculated from the block model using the nonlinear formulations given in (3)–(5).

At each iteration in the inversion scheme, (3)–(5) are approximated by a linear relation between the Bouguer gravity anomaly and the thickness of anomalous mass within a block:

$$g_k^{\text{calc}} = -G\Delta\rho \sum_{j=1}^{M} \frac{A_j b_j \cos \alpha_{kj}}{r_{kj}^2} \qquad k = 1, 2, \cdots, N \tag{8}$$

where b_j is the total thickness of the anomalous mass (Moho uplift plus mare fill) in the jth block, $\Delta\rho$ is the density contrast between the block and the surrounding crust (0.5 g/cm^3), A_j is the surface area of the jth block, $\mathbf{r}_{kj}$ is the vector between the kth observation point and the center of mass of all anomalous mass in the jth block, and α_{kj} is the angle at the observation point between $\mathbf{r}_{kj}$ and the downward vertical. From the linear relation (8) between g_k^{calc} and b_j it follows that $\Delta g_k = g_k^{\text{calc}} - g_k^{\text{obs}}$ can be linked to perturbations in the thickness of anomalous mass by the relationship

$$\Delta g_k = \sum_{j=1}^{M} \frac{\partial g_k^{\text{calc}}}{\partial b_j} \Delta b_j \tag{9}$$

or

$$\Delta \mathbf{g} = \mathbf{P}\, \Delta \mathbf{b} \tag{10}$$

where $\Delta \mathbf{g}$ is an $N \times 1$ column vector of residual gravity anomalies, $\mathbf{P}$ is an $N \times M$ matrix of partial derivatives ($P_{kj} = -G\, \Delta\rho\, A_j \cos \alpha_{kj}/r_{kj}^2$), and $\Delta \mathbf{b}$ is an $M \times 1$ vector of thickness corrections to the anomalous mass.

For $N > M$, (10) can be treated as a least squares problem and inverted by any number of methods [e.g., *Lawson and Hanson*, 1974] to obtain the corrections to the thickness of anomalous mass in each block. These corrections are then added to the mass thicknesses in the crustal model from the prior iteration, subject to the constraint of premare isostasy, imposed where applicable, to separate the contributions from Moho relief and mare fill. After each iteration the global datum D_M for Moho relief is adjusted to meet the constraint on crustal thickness inferred from seismic measurements for the region of the Apollo 12 and 14 landing sites. Finally, a new Bouguer anomaly field g^{calc} is computed from the adjusted model using (3)–(5), the rms residual gravity anomaly is determined from (7), and the inversion process is repeated until the rms residual converges to a minimum.

We impose one additional constraint on blocks in mare regions. If the Bouguer anomaly over a mare block is sufficiently small, the premare isostasy constraint may not be consistent with a finite thickness of basalt. Therefore, if the adjustment to the basalt thickness within a mare block forces the total mare thickness to fall below 250 m, the constraint of premare isostasy for that block is relaxed and the basalt thickness is held constant at 250 m for the remaining iterations.

A substantial savings in computation time and core storage was achieved by filling the matrix $\mathbf{P}$ only with those partial derivatives associated with the block directly below the observation point and the eight surrounding blocks. This approximation is equivalent to assuming that the residual anomaly above a given block is caused only by errors in the adopted thicknesses of the anomalous mass in the nine nearest blocks. The resulting partial derivative matrix is then sparse and can be easily manipulated into a diagonally dominant form. We solve for the vector $\Delta \mathbf{b}$ of thickness corrections by converting the matrix $\mathbf{P}$ to upper triangular form by using a series of Householder transformations applied to sequentially accumulated rows [*Lawson and Hanson*, 1974, pp. 207–311].

GRAVITY AND TOPOGRAPHIC DATA

The inversion procedure described above to determine crustal and upper mantle structure would be straightforward if the free air anomaly and topography were known everywhere on the moon within a well-prescribed uncertainty. Such, unfortunately, is not the case. Information on the lunar gravity field is derived from measurements of Doppler shifts in the frequency of radio transmissions along a line-of-sight (LOS) direction between the earth and a satellite in orbit about the moon. High-frequency variations in these Doppler observations contain a signature of gravity anomalies arising from near-surface heterogeneities in density. To extract this signature, the effect of orbital parameters on the LOS Doppler data must be removed or modeled. The estimation of a representation of the free air gravity field over a significant area of