TABLE 3. Principal Component Analysis for Telescopic
Reflectance Data

PCA Axis Number	Normalized Data (for Linear Model)		SSA Transformed Data (for Nonlinear Model)	
	Variance	% of Total	Variance	% of Total
1	$2.93e-5$	81.34	$2.72e-5$	93.47
2	$6.12e-6$	16.96	$1.74e-6$	5.97
3	$4.53e-7$	1.25	$1.06e-7$	0.36
4	$6.07e-8$	0.17	$2.90e-8$	0.10

and 1.8 μm were used to define each telescopic spectrum. Wavelengths near the telluric water bands were not sampled for this analysis (reducing the random residual errors in the PCA analysis to 0.5% reflectance). Since direct albedo information was not obtained from the telescopic data, all spectra were normalized by dividing each reflectance value by the sum of the reflectances at the 30 wavelengths.

Principal component analyses were performed on these modified spectra to describe the variation in the reflectance data and to test the types of mixing systematics that may account for the variations. A more detailed description of these procedures and strategy can be found in works by *Smith et al.* [1984] and *Johnson et al.* [1984]. Since PCA procedures are essentially linear transformations of the data, the linear mixing model can be readily applied to results from PCA of the normalized reflectance data. The nonlinear mixing model for intimate mixtures, however, first requires conversion of the reflectance measurements to single-scattering albedos (SSA) before initiation of PCA analysis. The normal albedo data of *Pohn and Wildey* [1970] were used to scale the telescopic spectra to the appropriate albedo value which then allows conversion to single-scattering albedo using the equations described by *Hapke* [1981]. This conversion is the major difference between how the data were treated for the linear checkerboard model and the nonlinear intimate mixture model. In order to allow the results of the two PCA analyses to be compared in the mixing models, the single scattering albedo spectra were renormalized (a linear function), thus removing albedo as a variable component before application of PCA.

The first few principal components of variation in the reflectance data describe most of the variation between spectra and allow the data to be examined for possible mixing systematics between end-member spectra. After choosing appropriate end-members to represent the data, the PCA variations in all spectra can be described as a proportional mixture of these end-members. To test how well the two mixing models describe the data, intermediate spectra are calculated from PCA end-member spectra using the proportions from the appropriate PCA. The calculated mixture spectra from the two models are compared to the real spectra, and the total residual error (RMS) of their difference is examined. This error should be minimized if the model describes the mixing systematics well.

Results

The PCA that utilized the nonlinear transformation to single-scattering albedos (for the intimate mixture model) contained a greater proportion of the variance within fewer orthogonal components. A comparison of the variance for the two PCA's performed on the telescopic data is shown in Table 3. Even before applying the mixing model analyses it is apparent that the nonlinear transformation PCA describes the variations of the data better than the linear PCA.

Results of the PCA analysis of the telescopic spectra after the single-scattering albedo conversion are shown in Figure 9. The first principal component of variation (axis 1) is largely sensitive to the overall slope of the reflectance spectra, while the second principal component (axis 2) is more sensitive to the absorption band depths near 1 μm. Note that if normalized spectra were not used in the PCA, the primary axis would be albedo.

The spectra represented in Figure 9 can be considered as mixtures of three end-members: mature highland soil at Copernicus crater (E5, F3, W3), mature soil on nonrayed mare (M1), and fresh mare crater ejecta (CM). It is reasonable to select E5, M1, and CM as spectral end-members for the purpose of analyzing mixing, as was previously inferred from inspection of the reflectance spectra and from measurement of the wavelength of the pyroxene absorption band near 1 μm. Figure 9 shows that all of the telescopic spectra for Copernicus's ray, except R6 and R7, can be considered to lie approximately along a mixing line between E5 and M1. Areas R6 and R7 appear to require a component of fresh mare material (CM), again in keeping with the conclusions drawn earlier based on the depths and wavelengths of the pyroxene bands.

The Copernicus data are consistent with compositional trends defined by a similar analysis of the spectra of about 500 lunar samples [*Smith et al.*, 1985; *Johnson et al.*, 1985]. The relationship between the telescopic and the laboratory data sets is illustrated in Figure 10. Samples presented in Figure 10 were selected to represent known major compositional and spectral classes identified from the lunar sample collection: noritic rocks (77017), mature anorthositic soils (61241), mare basalts (12063), and mature basaltic soils (12070). Apollo 12 basalt and soil were chosen because they may reasonably represent the low-Ti mare materials near Copernicus. All spectra in Figure 10 were filtered, recalculated as single-scattering albedos, renormalized, and rotated to the PCA vectors defined by the above four lunar samples. The telescopic spectra (Figure 10) thus plot in slightly different positions in relation to one another when the PCA is defined by the lunar samples.

When referenced to lunar samples, the Copernicus crater materials W3, F3, and E5 (Figure 1b) plot close to mature Apollo 16 soil (sample number 61241). Outer ejecta-blanket areas E3 and E4 and inner ray, sites R2 and R5 lie along a broad potential mixing line between the mature highland soil and mature mare soil (Sample number 12070). Mare material, M1, plots close to 12070. The spectra of R6 and R7, the most cratered of ray areas measured, fall below and to the right of M1 along a mixing line with CM. The M1-CM mixing line lies close to and parallel to the 12070-12063 mixing line. The more extensive collection of lunar sample spectra, therefore, provides a consistent and reasonable framework for interpretation of the telescopic spectra for Copernicus rays.

Since PCA is essentially a linear analysis of data variation, the position of any data measurement relative to a hypothetical mixing line between end-members describes the proportion of each member contributing to the measurement. As an example of mixing systematics that can be calculated from PCA results for Copernicus areas E5, M1A, and CM were chosen as end-members from the data presented in Figure 9. Listed in Table 4 are the calculated proportions of each end-member for the spectra along Copernicus's ray. Since area M1 was measured many times, the choice of M1A as an end-member to represent the M1 area allows the uncertainty of these proportions to be estimated as 5%.

The calculated fractions of these three end-members for