$\zeta(x, t)$ and $z = H + \varepsilon(x, t)$, respectively, where $x$ is the horizontal coordinate, $z$ is the vertical coordinate (positive downwards), and $t$ is time (see Figure A1). Let $\zeta$ and $\varepsilon$ be harmonic in $x$ at time $t = 0$:

$$\zeta(x, 0) = \zeta_0 \cos kx \qquad \varepsilon(x, 0) = \varepsilon_0 \cos kx \tag{A1}$$

where $k$ is the horizontal wave number.

We seek solutions for $\zeta$ and $\varepsilon$ of the form

$$\zeta(x, t) = F(t) \cos kx \qquad F(0) = \zeta_0$$
$$\varepsilon(x, t) = G(t) \cos kx \qquad G(0) = \varepsilon_0 \tag{A2}$$

The quantities $\varepsilon_0$ and $\zeta_0$ may be directly related by isostatic compensation prior to viscous relaxation. For local isostasy, the degree of isostatic compensation $c$ ($0 \le c \le 1$) is given by

$$c = -\Delta\rho\, \varepsilon_0/\rho\zeta_0 \tag{A3}$$

where $\Delta\rho = \rho_m - \rho$.

We assume that the viscous layer at $t > 0$ is always in a state of quasi-static equilibrium, so that the statement of momentum conservation is

$$\sigma_{ij,j} + f_i = 0 \qquad i, j = 1, 2, 3 \tag{A4}$$

where $\sigma_{ij}$ is a component of the stress tensor (positive in tension), $f_i$ is a component of the body force, $(1, 2, 3) = (x, y, z)$, the comma before the subscript denotes differentiation with respect to the subscripted coordinate, and repeated indices imply summation. In this problem, $f_x = f_y = 0$ and $f_z = \rho g$. The stress tensor is given by [e.g., *Fung*, 1977]

$$\sigma_{ij} = -p\delta_{ij} + 2\eta V_{ij} \tag{A5}$$

where $p$ is the pressure, $\delta_{ij}$ is the Kronecker delta, and

$$V_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) \tag{A6}$$

where $v_i$ is a component of fluid velocity. Because the problem is two-dimensional, $v_y = 0$ and $\partial/\partial y = 0$. Then the condition of incompressibility, $v_{kk} = 0$, is satisfied by

$$v_x = -\partial\psi/\partial z \qquad v_z = \partial\psi/\partial x \tag{A7}$$

where $\psi$ is a scalar stream function. From (A4)–(A7) we have the field equations

$$-\frac{\partial p}{\partial x} - \eta\left(\frac{\partial^3\psi}{\partial x^2\partial z} + \frac{\partial^3\psi}{\partial z^3}\right) = 0$$
$$-\frac{\partial p}{\partial z} + \eta\left(\frac{\partial^3\psi}{\partial x^3} + \frac{\partial^3\psi}{\partial x\partial z^2}\right) + \rho g = 0 \tag{A8}$$

There are six boundary conditions that must be specified to determine a solution to (A8). Two kinematic boundary conditions on $v_z(x, z, t)$ are

$$v_z(x, \zeta, t) = \dot{\zeta}(x, t) \qquad v_z(x, H + \varepsilon, t) = \dot{\varepsilon}(x, t) \tag{A9}$$

where the dot denotes $\partial/\partial t$. In addition there are four stress boundary conditions: the tangential component of stress must vanish at both the top and bottom of the viscous layer,

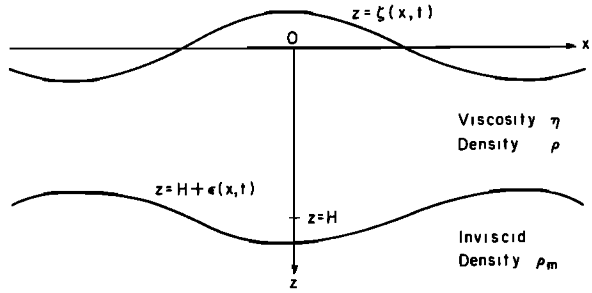


Fig. A1. Schematic view of viscous relaxation problem for a viscous layer over an inviscid half space of higher density. Parameters are defined in the text.

and the normal component of stress must be zero at the surface and $-\rho g H - \rho_m g \varepsilon(x, t)$ at the interface, where $g$ is gravitational acceleration. The condition at the interface reflects the hydrostatic state of the inviscid substratum. Note that these boundary conditions differ from those assumed by *Ramberg* [1968] in his treatment of this problem.

We assume that the solutions to (A8) are of the form

$$p(x, z, t) = \rho g z + P(z, t) \cos kx \qquad \psi(x, z, t) = \Psi(z, t) \sin kx \tag{A10}$$

Substitution of (A10) into (A8) gives

$$P = -k\eta \frac{\partial\Psi}{\partial z} + \frac{\eta}{k}\frac{\partial^3\Psi}{\partial z^3}$$
$$\frac{\partial^4\Psi}{\partial z^4} - 2k^2\frac{\partial^2\Psi}{\partial z^2} + k^4\Psi = 0 \tag{A11}$$

Equations (A11) have the solutions

$$\Psi(z, t) = \frac{\rho g}{2\eta k^3}[A(t)e^{-kz} + B(t)kze^{-kz} + C(t)e^{kz} + D(t)kze^{kz}]$$
$$P(z, t) = \frac{\rho g}{k}[B(t)e^{-kz} + D(t)e^{kz}] \tag{A12}$$

where $A$, $B$, $C$, and $D$ are dimensionless functions of time. Substitution of (A12) into (A10) gives

$$p(x, z, t) = \rho g z + \frac{\rho g}{k}[B(t)e^{-kz} + D(t)e^{kz}] \cos kx$$
$$\psi(x, z, t) = \frac{\rho g}{2\eta k^3}[A(t)e^{-kz} + B(t)kze^{-kz} + C(t)e^{kz} + D(t)kze^{kz}] \sin kx \tag{A13}$$

and the vertical velocity and stress components are given by

$$v_z = \frac{\rho g}{2\eta k^2}[Ae^{-kz} + Bkze^{-kz} + Ce^{kz} + Dkze^{kz}] \cos kx$$
$$\sigma_{xz} = -\frac{\rho g}{k}[(A - B)e^{-kz} + Bkze^{-kz} + (C + D)e^{kz} + Dkze^{kz}] \sin kx \tag{A14}$$