

ear algebra community, and many different algorithms have been proposed for their solution.

In this section, we briefly introduce a few of the most effective and relevant linear algebraic techniques. We begin in §2.4.1 by showing how the simple notion of a matrix splitting may be used to generate an entire family of iterative algorithms. Then, in §2.4.2, we briefly present the conjugate gradient iteration, which is generally considered to be the most effective method for solving sparse positive definite systems. We conclude by discussing stopping criteria for iterative methods. Note that none of the techniques presented in this section directly address the complementary inference problem of calculating entries of the error covariance matrix \widehat{P} .

2.4.1 Stationary Richardson Methods

In this section, we consider equation (2.46), assuming only that $A > 0$. The unique solution $x = A^{-1}b$ must clearly satisfy the following equation:

$$x = x + (b - Ax) \tag{2.47}$$

Given some initial guess x^0 , equation (2.47) naturally suggests the generation of a sequence of iterates $\{x^n\}_{n=1}^{\infty}$ according to the recursion

$$x^n = x^{n-1} + (b - Ax^{n-1}) = x^{n-1} + r^{n-1} = (I - A)x^{n-1} + b \tag{2.48}$$

where $r^n \triangleq (b - Ax^n)$ is the *residual* at the n^{th} iteration. This recursion is known as a *Richardson iteration* [46, 85]. It is stationary because the previous iterate is multiplied by the same matrix $(I - A)$ at each iteration.

The behavior of a linear system, like that defined by equation (2.48), is determined by its eigenvalues. Let the set of all eigenvalues of a matrix A be denoted by $\{\lambda_i(A)\}$. The spectral radius is then defined as $\rho(A) \triangleq \max_{\lambda \in \{\lambda_i(A)\}} |\lambda|$. It is well known that x^n will converge to the unique fixed point $x = A^{-1}b$, for arbitrary x^0 , if and only if $\rho(I - A) < 1$. The asymptotic convergence rate is equal to $\rho(I - A)$.