means. However, the error covariances are incorrect because the correlations induced by the cycles are not properly accounted for. Weiss and Freeman were also able to demonstrate a relationship between fast convergence and good approximations of the error covariances, and to provide a basic set of sufficient, but overly conservative, conditions guaranteeing the convergence of loopy BP.

More generally, researchers are beginning to develop a deeper theoretical understanding of loopy BP's behavior. In particular, it has been shown that belief propagation is intimately connected to the Bethe approximation of statistical physics [82, 83]. Loopy BP can be seen as minimizing a particular approximation to the relative entropy between the estimated beliefs and true pairwise marginal distributions [76]. The BP messages correspond to exponentiated sums of Lagrange multipliers that enforce the pairwise marginalization constraints (e.g. $\sum_{x_t} p(x_s, x_t) = p(x_s)$ for all $(s,t) \in \mathcal{E}$) which any locally consistent inference solution must satisfy [83, 84]. This connection has led to a much better understanding of the nature of the approximation made by loopy BP [76, 77], as well as a class of generalized belief propagation algorithms [83, 84] with superior performance.

2.4 Iterative Solution of Linear Systems

As shown by equation (2.6) of §2.1, the conditional mean $\widehat{x}$ of a Gaussian inference problem can be viewed as the solution of a linear system of equations. Thus, the problem of calculating $\widehat{x}$ is equivalent to the general linear algebraic problem of solving

$$Ax = b \tag{2.46}$$

for some symmetric positive definite matrix A. As discussed in §2.2.2, for the graphical inference problems examined in this thesis A will typically be sparse. Similar sparse, positive definite linear systems arise in many fields of science and engineering, perhaps most commonly from the discretization of elliptic partial differential equations [25]. For this reason, they have been widely studied in the numerical lin-