with mean μ and covariance $\mathcal{P}$, the information parameters are defined by

$$\vartheta = \mathcal{P}^{-1}\mu \qquad \Lambda = \mathcal{P}^{-1} \tag{2.28}$$

We use the notation $\mathcal{N}^{-1}(\vartheta, \Lambda)$ to indicate a Gaussian probability distribution with information parameters ϑ and Λ. For a more detailed introduction to information parameters, see Appendix B.1. We will denote the information parameters of the BP messages and conditional marginal distributions as

$$m_{ts}^{n}(x_s) = \alpha \mathcal{N}^{-1}(\vartheta_{ts}^{n}, \Lambda_{ts}^{n}) \qquad p(x_s \mid y_s^n) = \mathcal{N}^{-1}(\vartheta_s^n, \Lambda_s^n) \tag{2.29}$$

$$m_{ts}(x_s) = \alpha \mathcal{N}^{-1}(\vartheta_{ts}, \Lambda_{ts}) \qquad p(x_s \mid y) = \mathcal{N}^{-1}(\vartheta_s, \Lambda_s) \tag{2.30}$$

where equation (2.29) gives the values at iteration n, and equation (2.30) gives the corresponding steady–state values. The moment parameters of $p(x_s \mid y) \sim \mathcal{N}\left(\widehat{x}_s, \widehat{P}_s\right)$ are then related to the information parameters as follows:

$$\widehat{x}_s = (\Lambda_s)^{-1}\vartheta_s \qquad \widehat{P}_s = (\Lambda_s)^{-1} \tag{2.31}$$

Notice that Λ's are used to denote the inverse covariance parameters of messages and beliefs, while J's denote elements of the inverse covariance of the prior model.

To derive the Gaussian BP equations, we must determine the information parameterizations of all of the terms appearing in the general BP equations (2.25, 2.26). The pairwise clique potentials are already expressed in an information form by equation (2.15). Using Bayes' rule, we can write the local observation terms as

$$p(y_s \mid x_s) = \alpha \frac{p(x_s \mid y_s)}{p(x_s)} \tag{2.32}$$

Clearly, because $x \sim \mathcal{N}(0, P)$, $p(x_s) = \mathcal{N}^{-1}\big(0, P_{s,s}^{-1}\big)$. Combining the information form of the normal equations (2.6, 2.7) with the local measurement model (2.18), we find that $p(x_s \mid y_s) = \mathcal{N}^{-1}\big(C_s^T R_s^{-1} y_s, P_{s,s}^{-1} + C_s^T R_s^{-1} C_s\big)$. Then, taking the quotient