tent probability distribution.[4] However, the message–passing recursions discussed in §2.3 are valid even if the potential terms are not positive definite.

In the remainder of this thesis, we primarily use inverse covariance matrices $J = P^{-1}$ to parameterize Gaussian Markov random fields. Sometimes, however, it is useful to reparameterize the model in terms of clique potential functions (2.14, 2.15). In such cases, it is important to remember that the decomposition into clique potentials is *not* unique. Also, note that we do not employ the state space formalism traditionally used in the time series literature [44]. State space models correspond to the factorization of $p(x)$ into a product of an initial distribution $p(x_0)$ and a series of one–step transition distributions $p(x_t \mid x_{t-1})$. While such factorizations naturally extend to the partial ordering offered by tree–structured graphs [20], they cannot be consistently constructed for graphs with cycles.

2.3 Graph–based Inference Algorithms

In this section, we apply the graphical model formalism developed in §2.2 to the linear estimation problem introduced in §2.1. Assume that the unobserved random vector $x \sim \mathcal{N}(0, P)$ is a Gaussian process which is Markov with respect to an undirected graph $\mathcal{G}$. Because the variables $\{x_s\}_{s=1}^N$ are unobserved, their associated nodes in $\mathcal{G}$ are called *hidden* nodes.[5] As discussed in §2.2.3, $p(x)$ is parameterized by a *sparse* inverse covariance matrix $J = P^{-1}$. The observations y are assumed to be generated according to equation (2.3).

When the prior distribution $p(x)$ is parameterized by an inverse covariance matrix J, the conditional distribution $p(x \mid y) \sim \mathcal{N}\left(\widehat{x}, \widehat{P}\right)$ is naturally expressed as the solution of the information form of the normal equations (2.6, 2.7). We assume, without loss of generality,[6] that the observation vector y decomposes into a

[4] There exist graphs with cycles for which *no* factorization will have normalizable clique potentials.

[5] In the recursive estimation literature, x_s is the *state* variable associated with node s [20, 44]. However, this terminology is not entirely appropriate for graphs with cycles because $\{x_t | t \in N(s)\}$ are not, in general, conditionally independent given x_s.

[6] Any observation involving multiple hidden nodes may be represented by a clique potential which includes those nodes. For Gaussian graphical models, such potentials simply modify certain entries