of a combination of local observations and interactions. This local structure allows complicated stochastic processes to be specified very compactly. In addition, efficient inference algorithms can exploit this structure to achieve huge savings over direct computational costs. For these reasons, graphical models have been widely applied in such fields as artificial intelligence [59], error correcting codes [34, 51], speech processing [60], statistical physics [82], image processing [36, 66], remote sensing [24, 29], and computer vision [28, 32, 50].

2.2.1 Graph Separation and Conditional Independence

A graph $\mathcal{G}$ consists of a set of nodes or vertices $\mathcal{V}$, and a corresponding set of edges $\mathcal{E}$. Graphical models associate each node $s \in \mathcal{V}$ with a random vector x_s. In general, x_s could be drawn from a wide variety of probability distributions. In this thesis, however, we focus on the case where x_s is a Gaussian random vector of dimension $d_s \triangleq \dim x_s$. For any subset $\mathcal{A} \subset \mathcal{V}$, we denote the set of random variables in $\mathcal{A}$ by $x_{\mathcal{A}} \triangleq \{x_s\}_{s\in\mathcal{A}}$. If the $N \triangleq |\mathcal{V}|$ nodes are indexed by the integers $\mathcal{V} = \{1, 2, \ldots, N\}$, the Gaussian stochastic process defined on the overall graph is given by $x \triangleq \begin{bmatrix} x_1^T & x_2^T & \cdots & x_N^T \end{bmatrix}^T$. Note that $\dim x = \sum_{s\in\mathcal{V}} d_s$.

Graphical models use edges to implicitly specify a set of conditional independencies. Each edge $(s, t) \in \mathcal{E}$ connects two nodes $s, t \in \mathcal{V}$, where $s \neq t$. In this thesis, we exclusively employ *undirected* graphical models for which the edges (s, t) and (t, s) are equivalent.[2] Figure 2-1(a) shows an example of an undirected graphical model representing five different random variables. Such models are also known as *Markov random fields* (MRFs), or for the special case of jointly Gaussian random variables as *covariance selection models* in the statistics literature [26, 48, 69]. For a graphical interpretation of many standard Gaussian data analysis techniques, see [61].

When describing the statistical properties of Markov random fields, the structural properties of the underlying graph play an important role. A *path* between nodes s_0

[2]There is another formalism for associating conditional independencies with graphs which uses directed edges. Any directed graphical model may be converted into an equivalent undirected model, although some structure may be lost in the process [48, 59].