Chapter 2

Background

In this thesis, we develop computationally efficient algorithms for solving Gaussian statistical inference problems defined by *graphical models*. The primary purpose of this chapter is to provide a self–contained introduction to graphical models, emphasizing their powerful ability to express globally consistent probability distributions through a series of local constraints. After a brief review of linear estimation in §2.1, we demonstrate in §2.2 how graphs are associated with probability distributions, focusing on the particular features of Gaussian models. Then, in §2.3, we discuss the strengths and weaknesses of existing inference algorithms for graphical models. In particular, we provide a detailed derivation of a popular inference algorithm known as *belief propagation* which forms the basis for many of the new results in this thesis. The chapter concludes in §2.4 with an introduction to some relevant techniques from the numerical linear algebra literature.

2.1 Linear Estimation

This thesis focuses on a classical problem in estimation theory. We are given a vector of observations y of an unknown random vector x, where x and y are jointly Gaussian with zero mean.[1] Under these assumptions, the conditional distribution

[1]Non–zero means can be easily accounted for by estimating the deviation from the mean using the modified random variables $\bar{x} = (x - E[x])$ and $\bar{y} = (y - E[y])$.