Chapter 3, Embedded Trees

Chapter 3 presents a family of tree–based inference algorithms for solving estimation problems defined on graphs with cycles. The chapter begins with a discussion of the connections between spanning tree distributions and graphs with cycles. Then, we use these connections to develop the embedded trees (ET) algorithm, which computes a sequence of estimates converging to the conditional mean of the loopy estimation problem. A combination of theoretical results and numerical examples are used to demonstrate several properties of the ET iteration, including the important performance improvements obtained by employing multiple embedded trees.

Following the discussion of the ET iteration for estimates, we present a complementary tree–based iterative algorithm for calculating error variances. The computational complexity of this algorithm is proportional to the number of edges which must be removed from the graph with cycles to reveal an embedded tree. Thus, it will be particularly efficient for the very sparse loopy graphs discussed in §1.1. We present several theoretical results which may be used to guarantee the convergence of either inference algorithm. The chapter concludes with a set of simulations demonstrating the effectiveness of the proposed methods.

Chapter 4, Accelerated Tree–Based Iterations

In Chapter 4, we present several techniques which use the basic embedded trees iteration developed in Chapter 3 to create more rapidly convergent algorithms. We start by demonstrating that the standard ET iteration may sometimes be made to converge faster by appropriately modifying the numerical structure of the tree–structured subproblems solved at each iteration. Then, we show that the dynamics of the ET iteration reveal information about the directions in which the errors in the current estimate lie. This leads to a rank–one correction technique which first estimates the dominant error direction, and then removes those errors on subsequent iterations.

We discuss certain inefficiencies with the rank–one acceleration method, which in turn motivate the use of the ET algorithm as a preconditioner for the conjugate