pairs of fine–scale nodes where discontinuities are likely to arise. Such edges should be able to account for short–range dependencies neglected by standard multiscale models. To illustrate this idea, we consider the modeling of the one–dimensional process of length 32 whose exact covariance P is shown in Figure 1-2(a). We approximate this process using two different graphical models: a multiscale tree (Figure 1-2(b)), and a "near–tree" containing an additional edge across the largest fine scale tree boundary (Figure 1-2(c)). In both models, the dimension of the Gaussian variable at each node is constrained to be 2; therefore, the finest scale contains 16 nodes to model all 32 process points.

The tree–based multiscale model was realized using the scale–recursive algorithm presented in [30, 31]. Figure 1-2(d) shows the resulting fine–scale covariance matrix P_{tree}, while Figure 1-2(f) gives the corresponding absolute error $|P - P_{\text{tree}}|$. The tree model matches the desired process statistics relatively well except at the center, where the tree structure causes a boundary artifact. In contrast, Figures 1-2(e) and 1-2(g) show the covariance P_{loop} and absolute error $|P - P_{\text{loop}}|$ which can be attained by the augmented multiscale model. The addition of a single edge has reduced the peak error by 60%, a substantial gain in modeling fidelity. In §3.5.2, we show that the inference algorithms developed in this thesis can efficiently and accurately calculate both conditional means and error variances for this model.

The previous example suggests that very sparsely connected graphs with cycles may offer significant modeling advantages relative to their tree–structured counterparts. Unfortunately, as the resulting graphs do have cycles, the extremely efficient inference algorithms which made tree–structured multiscale models so attractive are not available. The primary goal of this thesis is to develop novel inference techniques which allow estimates, and the associated error variances, to be quickly calculated for the widest possible class of graphs. The algorithms we develop are particularly effective for graphs, like that presented in this example, which are nearly tree–structured. We hope that our results will motivate future studies exploring the design of multiscale models defined on graphs with cycles.