Embedded Trees: Estimation of Gaussian Processes on Graphs with Cycles

by

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Abstract

Structured statistical models play a critical role in controlling the complexity of large-scale estimation problems. Graphical models provide a powerful general framework for encoding this structure. For graphs which are acyclic or tree-structured, there exist efficient exact algorithms for statistical inference. However, real-world phenomena are often only well modeled by graphs with cycles. As direct estimation procedures are generally prohibitively costly for such graphs, it is necessary to develop more efficient iterative alternatives.

In this thesis, we develop a novel iterative estimation algorithm for Gaussian processes defined on graphs with cycles. It operates by exactly solving a series of modified estimation problems on spanning trees embedded within the original cyclic graph. When the algorithm converges, it always computes the correct conditional means. In contrast to many other iterative estimation algorithms, the tree-based procedures we propose may be extended to calculate exact error variances. Although the conditional mean iteration is effective for quite densely connected graphical models, the error variance computation is most efficient for sparser graphs. In this context, we present a modeling example which suggests that very sparsely connected graphs with cycles may provide significant advantages relative to their tree-structured counterparts.

The convergence properties of the proposed tree-based iterations are extensively characterized both analytically and experimentally. We also provide an analysis of the geometric structure underlying these iterations which naturally suggests techniques for accelerating their convergence. Several different acceleration methods are proposed and analyzed, the most effective of which uses the tree-based iterations to precondition the conjugate gradient method. Simulation results are presented showing that for many problems, accelerated tree-based iterations converge much more rapidly than competing techniques.

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