Note the similarity of this expression with part of the updates in Equation 12. By caching the necessary statistics needed to update $\theta$, we can calculate our ELBO in an efficient manner.

### A.3 Updates for the global stick-breaking weights $\beta$

The global stick breaking weights $\beta$ is not conjugate to node membership weights $\pi$. In order to obtain point estimates for $\beta$ we perform a two-metric constrained optimization using its first order gradients. We can write the objective for $\beta$ w.r.t to our ELBO in the following manner:

$$\mathcal{L}(\beta) = \sum_{k=1}^{K}(\gamma - 1)\log(1 - v_k) - N\sum_{k=1}^{K}\log\Gamma(\alpha\beta_k) + \sum_{k=1}^{K}(\alpha\beta_k - 1)\sum_{i=1}^{N}\mathbb{E}_q[\log\pi_{ik}] \tag{9}$$

Since $\beta_k = v_k\prod_{\ell=1}^{k-1}(1 - v_\ell)$ and $\beta_{K+} = 1 - \sum_{k=1}^{K}\beta_k$, we redefine the prior over $\beta$ as a sum over independently distributed beta variables $v$. We can obtain point estimates for $v$ without having to worry about the constrained optimization task for $\beta$ which is significantly more costly then the two-metric constrained optimization over $v$. We now take the derivatives for $\beta$ with this in mind:

$$\frac{d\mathcal{L}(\beta)}{dv_m} = \frac{-(\gamma-1)}{(1-v_m)} - \alpha N\sum_{k=1}^{K}\frac{d\beta_k}{dv_m}\psi(\alpha\beta_k) + \alpha\sum_{k=1}^{K}\frac{d\beta_k}{dv_m}\sum_{i=1}^{N}\mathbb{E}_q[\log\pi_{ik}] \tag{10}$$

where the derivative $\frac{d\beta_k}{dv_m}$ will change depending on the value of $k$. When $m > k$, then $\frac{d\beta_k}{dv_m} = 0$. When $m = k$, then $\frac{d\beta_k}{dv_m} = \frac{\beta_k}{v_m}$. Finally, when $m < k$, then $\frac{d\beta_k}{dv_m} = \frac{-\beta_k}{(1-v_m)}$.

Our constrained optimization provides us with updates for $v*$ at iteration $t$ which we can then use in our stochastic variational approach by setting $v_k^t = (1 - \rho_t)v_k^{t-1} + \rho_t(v_k^*)$. From this we can determine a new set of values for $\beta^t$ by setting $\beta_k^t = v_k^t\prod_{\ell=1}^{k-1}(1 - v_\ell^t)$.