6. Conclusion

To calculate the size and location of shallow landslides, we propose conceiving of landscapes as clusters of small cells with properties (e.g., soil depth, root strength, and pore pressure) that influence the forces driving their failure and the forces resisting failure that they exert on one another. The potential hazard and geomorphic significance of shallow landslides depend on their location and size. Commonly applied one-dimensional stability models do not include lateral effects and cannot predict landslide size. Multidimensional models must be applied to specific geometries that are typically not known a priori, and testing all possible geometries is computationally prohibitive. We present an efficient deterministic search algorithm based on spectral graph theory and couple it with a multidimensional stability model to predict discrete landslides in catchment-scale applications using gridded spatial data. The algorithm is general, assuming only that instability results when driving forces acting on a cluster of cells exceed the resisting forces on its margins and that clusters behave as rigid blocks with a failure plane at the soil-bedrock interface. When applied to a synthetic landscape with predefined regularly and irregularly shaped reduced-strength unstable patches, the algorithm recovers landslides with shape and size similar to these patches. When applied to an intensely investigated field site near Coos Bay, Oregon (CB-1), the algorithm predicts the size and location of an observed shallow landslide using field-measured physical parameters. While predictions of location and shape are robust to modest variations in input parameters, size is more sensitive, particularly to pore pressure variations. In these applications, the search algorithm identifies patches of potential instability within large areas of stable landscape. Within these patches will be many different combinations of cells with Factor of Safety less than one, suggesting that subtle variations in local conditions may determine the ultimate form and exact location at a specific site. Nonetheless, the preliminary tests presented here suggest that the search algorithm enables the predictions of shallow landslide locations, sizes, and shapes across landscapes.

Our results suggest that in natural landscapes prone to landsliding, the procedure identifies large areas of stable landscape and patches of potential instability. Within these patches, many different combinations of cells are found with FS less than unity, implying that subtle variations in local conditions determine the form and location of failure at a specific site. This implies that, in the absence of high-resolution strength and resistance data, calibration and testing of the model should be based on the frequency distribution of sizes and locations of the predicted and observed landslides rather than on prediction of exact location of individual slides. This new search algorithm enables the prediction of shallow landslide size and locations across landscapes. However, application of our method to a larger landscape will involve making choices on how to parameterize landslide-relevant spatial properties at sufficiently fine resolution. This is the subject of subsequent research presented in D. Bellugi et al. (submitted).

Appendix A: Computational Complexity and Implementation

A naïve approach that tests every combination of grid cells requires a number of operations that grows exponentially with the number of grid cells. In contrast, here we show that the number of operations required by our procedure grows quadratically with the number of grid cells and linearly with the number of eigenvectors examined. This appendix also presents implementation details, including the parallelization of the search algorithm.

Iterative algorithms for numerical computation of eigenvectors and eigenvalues, typically based on the Lanczos algorithm [*Lanczos*, 1950], proceed by multiplying the target matrix by a series of vectors and are thus dominated by matrix-vector multiplication [*Demmel et al.*, 2007]. Due to the sparsity of our Laplacian-like resistance matrices which have at most five nonzero entries in each row (Figure 3c), the cost of one sparse matrix-vector multiplication scales linearly with the number of grid cells. The number of Lanczos iterations is upper bounded by the matrix dimension n, so in the worst case the number of operations required to compute k eigenvectors is kn^2. In practice the Lanczos method often converges to numerical precision after many fewer than n iterations, and thus, running time is significantly lower than the kn^2 bound.

The cost of finding connected components in a graph is linear in the number of nodes and edges [*Tarjan*, 1972], but the process is repeated for n thresholds of k eigenvectors, resulting in an upper bound of kn^2 operations. Similarly, comparing at each threshold the current and previous connected components to determine which regions overlap can be accomplished via a loop which examines each spatial cell once, also

