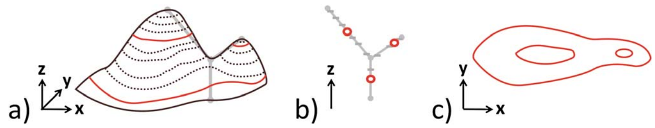

Figure 4. (a) Side view of the 3-D surface defined by the continuous vector **x***. Dashed contour lines define regions r_i associated with threshold t_i, solid contour lines indicate the regions with the minimal FS for each gray edge in the contour tree T_c, with solid gray circles representing the local minima and maxima of **x***. (b) The contour tree T_c arising from **x***. Solid gray circles are the vertices of T_c, corresponding to the birth, and merging of the regions r_i; solid gray lines correspond to the edges of T_c in which individual regions grow; dashes represent the thresholds t_i; unfilled circles show the thresholds that minimize the FS for each edge. (c) Overhead view of Figure 4a showing only the three overlapping regions r_i from thresholds t_i that minimize the FS for each edge of T_c, corresponding to the solid red lines in Figure 4a and the unfilled red circles in Figure 4b.

3.3. Recovering Discrete Landslides

An approximation to the discrete solution **x** of equation (10) can be obtained by thresholding the continuous vector **x***. As **x*** is also of length *nm*, all $nm - 1$ possible interesting threshold values t_i are examined. All $\mathbf{x_i}$ for which the corresponding $\mathbf{x_i}$* are greater than t_i are set to 1, and the rest are set to 0. In other words, **x*** can be visualized as a surface with each threshold defining a contour line on the surface, and the regions at threshold t_i are the projection on the Cartesian plane of the points with values higher than t_i (Figure 4a). These regions are extracted using connected-component analysis [*Haralick and Shapiro*, 1992], by which any threshold t_i may give rise to disconnected discrete regions r_i in which $r_i = 1$ if $\mathbf{x_i}^* > t_i$ and $r_i = 0$ otherwise. In particular, a region r_i at threshold t_i must either have not existed at threshold t_{i-1}, or be an expansion of a region r_{i-1}' at threshold t_{i-1}, or be the result of the merging of regions r_{i-1}' and r_{i-1}'' at threshold t_{i-1}. The evolution of the regions as the threshold is varied gives rise to a topological contour tree T_c (Figure 4b) in which vertices represent the birth and merging of regions, while edges represent the expansion or contraction of an individual region [*Freeman and Morse*, 1967; *Carr et al.*, 2000].

As we are interested not only in the least stable cluster of cells in a landscape but in all the unstable clusters, we examine the complete contour trees resulting from each eigenvector. Each edge in T_c is traversed, and at each threshold the FS of r_i is computed for the corresponding collection of cells. This allows the use of a vector sum to correctly compute the driving forces. On each edge, multiple thresholds may result in shapes that have a FS below unity (Figures 4a and 4b). These shapes will overlap one another (Figure 4c), but only one will actually fail at a specific location. There are many ways to choose among these unstable shapes. Here we choose the unstable shape with the lowest unstable FS, (referred to as FS_{min}) as it is most consistent with the optimization and is commonly assumed to be the most probable [e.g., *Montgomery and Dietrich*, 1994; *Rosso et al.*, 2006; *Stark and Guzzetti*, 2009]. However, it is also the case that under evolving conditions the first cluster of cells to cross the stability threshold can be the one that fails [*Casadei et al.*, 2003a]. Thus, the algorithm offers the option of retaining the overlapping shape with FS closest to but less than unity (i.e., the highest unstable FS, referred to as FS_{max}).

The thresholding process is then repeated for the complement of **x***, **x***′ (**x***′ = −**x***), which is also an eigenvector of equation (12), after the change of variables. This is equivalent to inverting the **x*** surface and generating a new contour tree T_c'. Thresholding **x***′ results in the complements of the discrete regions arising from **x*** (i.e., $r_i = 0$ if $\mathbf{x_i}^* > t_i$, and 1 otherwise), thus allowing the exploration of all the regions initiating from the local maxima and minima of **x*** (the local maxima of **x*** are the local minima of **x***′). The set of all regions extracted from T_c and T_c' represent the predicted landslides from the eigenvector **y***. This process is repeated for each of the *k* eigenvectors. While only one landslide is retained per edge of T_c and T_c', multiple landslide predictions arise from the many thresholds applied to many eigenvectors, and these landslides may overlap (Figure 4 illustrates the case of overlapping predictions from a single eigenvector). As only one of these overlapping landslides will fail for a particular storm event, they are pruned a posteriori using the same criterion (FS_{min} or FS_{max}) as in the thresholding process. In the FS_{min} case, predicted shapes are sorted by their FS in ascending order into a list. The first element of the list is pairwise compared to all the other elements in sorted order, and whenever another element of the list overlaps with the first it is removed from the list. The process is then repeated for the next element of the list until the list is fully traversed. The pruning process is similar in the FS_{max} case, but with the predicted shapes sorted in descending order.