upslope cell from its downslope neighbor can be different, as the passive resistance of the downslope cell generally is greater than the active force from the upslope cell. To obtain an undirected graph, the boundary forces are first symmetrized: the off-diagonal entries of **R**, visualized in Figure 3c, contain the negated average of the weights along a given edge:

$$\mathbf{R_{ij}} = \mathbf{R_{ji}} = -\frac{w_{ij} + w_{ji}}{2}, i \neq j. \quad (9)$$

While this formulation is rather different from the usual spectral clustering definitions [e.g., *Von Luxburg*, 2007], it allows the exact computation of the numerator of equation (6) while retaining a symmetric matrix. The matrices **R** and **F** have a similar form to the weighted adjacency (**A**) and degree (**D**) matrices of a graph, and $\mathbf{F}^{-1/2}\mathrm{R}\mathrm{F}^{-\mathbf{1/2}}$ is thus similar in form to the normalized Laplacian ($\mathbf{L} = \mathbf{D} - \mathbf{A}$) matrix of a weighted graph [*Chung*, 1997], which has the form $\mathbf{D}^{-\mathbf{1/2}}\mathrm{L}\mathrm{D}^{-\mathbf{1/2}}$ and is commonly used in spectral clustering algorithms [*Von Luxburg*, 2007].

The aim is to partition G to delineate unstable clusters of cells $S \in G$ (Figure 3a), defined by a binary indicator vector **x** of length nm. Each component $\mathbf{x}_k \in \{0,1\}$ indicates whether vertex v_k is part of S: $\mathbf{x}_k = 1$ if $v_k \in S$, and $\mathbf{x}_k = 0$ if $v_k \notin S$. The index k corresponds to the position of a grid cell in a linearized representation of the grid (Figure 3d). We define the cost function $C(\mathbf{x})$ of the partition S as the FS of S expressed in terms of **x**, **R**, and **F**:

$$C(\mathbf{x}) = \frac{\mathbf{x^T R x}}{\mathbf{x^T F x}}, \quad (10)$$

where $\mathbf{x^T}$ is the transpose of **x**. Consistent with the assumption of a rigid block with resistive forces acting on its margins, entries $\mathbf{R_{ji}}$ from equation (9), corresponding to the resistance between a pair of adjacent vertices v_i and v_j, are considered only if one of v_i and v_j corresponds to a nonzero entry of **x**. This is equivalent to assuming that lateral resistance acts between adjacent grid cells i and j if and only if they straddle the margin of the block. Similarly, entries $\mathbf{R_{ii}}$ from equation (8), which include the resistance on the base of v_i, are only considered if v_i corresponds to a nonzero entry of **x**. This is consistent with grid cell i contributing to the resistance on the base of the block if and only if it is part of the block. With this construction, averaging of the weights w_{ij} and w_{ji} in equation (9) does not change the cost function: $C(\mathbf{x})$ in equation (10) is exactly equal to FS in equation (6) for any partition defined by an indicator vector **x**.

The optimal partition S^* (the one having the lowest FS) is determined by the indicator vector $\mathbf{x}^*$ which minimizes $C(\mathbf{x}^*)$. Finding $\mathbf{x}^*$ in the discrete case (i.e., $\mathbf{x}^* \in \{0,1\}$) is an intractable problem [see *Shi and Malik*, 2000, Appendix A], and thus, an approximation must be used. Following an approach similar to *Shi and Malik* [2000], we find an approximate solution by relaxing the condition that $\mathbf{x}^* \in \{0,1\}$, and let $\mathbf{x}^*$ take real values (i.e., $\mathbf{x}^* \in \mathbb{R}$) (commonly referred to as a spectral relaxation). While this allows a tractable minimization of $\mathbf{x}^*$, the solution is no longer discrete (i.e., landslide or not landslide). Instead, $\mathbf{x}^*$ can be interpreted as a continuous indicator vector, representing the fraction of each node that contributes to the optimal partition S^*. If we let $\mathbf{y} = \mathbf{F}^{\mathbf{1/2}}\mathrm{x}$, and substitute into equation (10), the cost function becomes

$$C(\mathbf{y}) = \frac{\mathbf{y^T F^{-1/2} R F^{-1/2} y}}{\mathbf{y^T y}}. \quad (11)$$

The cost function defined by equation (11) has the same form as the Rayleigh quotient of a positive semidefinite matrix **M** and a nonzero vector **x**, defined as $(\mathbf{x^T M x})/(\mathbf{x^T x})$ [*Horn and Johnson*, 1985]: as **R** is a real-valued symmetric square matrix and **F** is a diagonal matrix with positive entries, the inner product of $\mathbf{F}^{-\mathbf{1/2}}\mathrm{R}\mathrm{F}^{-\mathbf{1/2}}$ is also symmetric and positive semidefinite. The minimizing solution to the Raleigh quotient is the eigenvector $\mathbf{y}^*$ pointed to by the smallest nonzero eigenvalue of the linear system defined by

$$\mathbf{F^{-1/2} R F^{-1/2} y} = \lambda \mathbf{y} \quad (12)$$

[*Horn and Johnson*, 1985]. The optimal partition S^* is thus defined by $\mathbf{x}^* = \mathbf{F}^{-\mathbf{1/2}}\mathrm{y}^*$, after reversing the change of variable $\mathbf{y} = \mathbf{F}^{\mathbf{1/2}}\mathrm{x}$ used to obtain the Rayleigh quotient. As $\mathbf{x}^*$ is only an approximation of the original discrete **x**, we do not limit ourselves to the first eigenvector, rather we take the first k eigenvectors, where k can be limited by a maximal value of $C(\mathbf{x}^*)$ or, in practice, by computational constraints. These eigenvectors effectively guide the search algorithm in finding patches of instability in the landscape. In this study we set $k = 164$, a compromise between a value that results in fast running times and one that results in a more exhaustive search.