

of other driving parameters (e.g., slope) and is not examined in detail here. In a related study, Bellugi et al. (submitted) find that, as a result of grid discretization, the perimeter length of 34 landslides mapped over a 10 year period in the area surrounding our study site can increase up to 140%. Because discretized area instead does not vary significantly, the estimation of driving forces is not affected. As a result, the stability of these mapped landslides tends to be overestimated (Bellugi et al., submitted).

The Factor of Safety (FS) for the group of columns is defined as the ratio of its total resisting to driving forces,

$$FS = \frac{\sum_{\text{all}} R_b + \sum_{\text{left}} R_l + \sum_{\text{right}} R_r + \sum_{\text{down}} R_d - \sum_{\text{up}} R_u}{\sum_{\text{all}} F_d}, \quad (6)$$

where the summation subscripts indicate whether the sums are performed over all the columns, or those on the left, right, downslope, or upslope margins only.

### 3. Search Algorithm

#### 3.1. General Framework

In a landscape such as that illustrated in Figure 1, a shallow landslide can be conceptualized as a collection of surface elements (e.g., grid cells), which behave coherently as a solid block, mobilizing together in accordance to their physical properties. We assume that the resolution of the grid is sufficiently fine to represent a landslide as a collection of grid cells, i.e., that it is finer than the smallest landslide of interest. While the effect of grid resolution is not explored in this manuscript, it is apparent that a resolution that is too coarse will result in smaller landslides not being predicted, and a resolution that is too fine will result in unnecessary computational burden. In practice, a grid resolution of 1–2 m results in a good balance between these two end-member cases. Our aim is to find all unstable collections of grid cells, but the exact solution to this problem requires testing the stability of every possible combination of grid cells. The number of possible combinations of cells grows exponentially with the number of cells  $n$  in the grid, resulting in up to  $2^n$  different combinations of cells. For example, exhaustively testing every combination of grid cells across a small 1 km<sup>2</sup> landscape discretized into a square grid composed of 1000 by 1000 cells (as one would obtain using modern lidar data) could result in exploring up to  $2^{1,000,000}$  combinations of cells, a number so vast that the task would be unfeasible even using the world's most powerful computers [Dasgupta et al., 2006].

Alternatively, the collection of cells that fails when its cumulative driving forces exceed the cumulative resistive forces (the landslide) could be thought of as the result of an optimization process that minimizes the ratio of resistive to driving forces (the FS). While no algorithm currently exists for finding exact solutions for this class of optimization problems in a polynomial number of iterations, clustering algorithms based on graph theory can provide good approximations. Graphs are combinatorial mathematical structures used to model pairwise relations between objects, consisting of a set of vertices (or nodes) that represent the objects, and a set of edges (or lines) that represent their relation or connection. Vertices and edges may have additional attributes that describe objects and their connections. If connections are asymmetric (e.g., the flow between two locations on a river), edges have directions and the graph is directed. If instead the connections are symmetrical (e.g., the line of sight between two mountaintops), edges have no direction and the graph is undirected. For more formal definitions we refer to common graph theory textbooks [e.g., Diestel, 2005; Gross and Yellen, 2005]. Graphs can also be represented using matrices, in which entries  $(i, i)$  encode the attributes of vertex  $i$  and entries  $(i, j)$  encode the attributes of the edge connecting vertex  $i$  and vertex  $j$ . In recent years spectral graph theory, the study of the properties of a graph in relationship to its associated matrices and their eigenvectors and eigenvalues (the graph's spectrum) [Chung, 1997], has enabled the use of modern linear algebra techniques in graph theory applications. In particular, this has allowed the development of algorithms that, by making use of graph spectra, can efficiently solve clustering problems [Von Luxburg, 2007]. Clustering is the task of grouping a set of objects such that those in the same group (a cluster) are more similar (in some sense or another) to each other than to those in other groups [Kaufman and Rousseeuw, 2005]. A gridded landscape can be depicted as a graph, with vertices corresponding to soil columns and edges to the forces that can develop between them. A cluster (e.g., of unstable cells) can be obtained by cutting all the edges that link those vertices (cells) inside the cluster (the landslide) with those outside (the stable landscape). For example, the four cells outlined in red in Figure 2a correspond to the four blue vertices of the graph