Bayesian Nonparametric Inference of Switching Dynamic Linear Models

Emily Fox, *Member, IEEE*, Erik B. Sudderth, *Member, IEEE*, Michael I. Jordan, *Fellow, IEEE*, and Alan S. Willsky, *Fellow, IEEE*

***Abstract*—Many complex dynamical phenomena can be effectively modeled by a system that switches among a set of conditionally linear dynamical modes. We consider two such models: the switching linear dynamical system (SLDS) and the switching vector autoregressive (VAR) process. Our Bayesian nonparametric approach utilizes a hierarchical Dirichlet process prior to learn an unknown number of persistent, smooth dynamical modes. We additionally employ automatic relevance determination to infer a sparse set of dynamic dependencies allowing us to learn SLDS with varying state dimension or switching VAR processes with varying autoregressive order. We develop a sampling algorithm that combines a truncated approximation to the Dirichlet process with efficient joint sampling of the mode and state sequences. The utility and flexibility of our model are demonstrated on synthetic data, sequences of dancing honey bees, the IBOVESPA stock index and a maneuvering target tracking application.**

***Index Terms*—Autoregressive processes, Bayesian methods, hidden Markov models, state-space methods, time series analysis, unsupervised learning.**

I. Introduction

LINEAR dynamical systems (LDSs) are useful in describing dynamical phenomena as diverse as human motion [3], [4], financial time-series [5]–[7], maneuvering targets [8], [9] and the dance of honey bees [10]. However, such phenomena often exhibit structural changes over time and the LDS models which describe them must also change. For example, a ballistic missile makes an evasive maneuver; a country experiences a recession, a central bank intervention, or some national or global event; a honey bee changes from a *waggle* to a *turn right* dance. Some of these changes will appear frequently, while others are only rarely observed. In addition, there is always the possibility of a new, previously unseen dynamical behavior. These considerations motivate us to develop a Bayesian nonparametric approach for learning *switching* LDS (SLDS) models. We also consider a special case of the SLDS—the switching vector autoregressive (VAR) model—in which direct observations of the underlying dynamical process are assumed available.

One can view the SLDS and the simpler switching VAR process, as an extension of hidden Markov models (HMMs) in which each HMM state, or *mode*, is associated with a linear dynamical process. Within the signal processing community, such HMM-based models have received considerable attention and proven useful in modeling the complex time evolution of signals. Specifically, HMMs have a long history of signal processing applications, with major success stories in speech processing (see the early influential tutorial by Rabiner [11]). While the HMM makes a strong Markovian assumption that observations are conditionally independent given the mode, the SLDS and switching VAR processes are able to capture more complex temporal dependencies often present in real data. Applications of switching linear dynamical processes, with roots in the control and econometrics literature, have recently become more prevalent within signal processing [10], [12]–[14]. However, most existing methods for learning SLDS and switching VAR processes rely on either fixing the number of HMM modes, such as in the preceding papers, or considering a change-point detection formulation where each inferred change is to a new, previously unseen dynamical mode, such as in [15]. There is growing interest in expanding the modeling framework to remove the purely parametric assumption of these previous formulations. In this paper we show how one can, in a seamless manner, remain agnostic about the number of dynamical modes while still allowing for returns to previously exhibited dynamical behaviors.

The rapidly developing field of *Bayesian nonparametrics* provides a new direction for analyzing HMMs with unknown state space cardinality. In particular, it has been shown that the hierarchical Dirichlet process (HDP) provides a useful prior on the HMM parameters [16], [17]. An alternative formulation of a Bayesian nonparametric HMM with application to music analysis has been presented in [18], though without the shared sparsity induced by the HDP. Another application of Bayesian nonparametrics to music analysis was presented in [19], where the authors propose Dirichlet process clustering of fixed-length segments of a time series, with each cluster modeling the dynamics of the given segments via a different finite HMM.

Manuscript received April 01, 2010; revised October 19, 2010; accepted December 04, 2010. Date of publication January 06, 2011; date of current version March 09, 2011. The associate editor coordinating the review of this manuscript and approving it for publication was Jerome Idier. This work was supported in part by MURIs funded through AFOSR Grant FA9550-06-1-0324 and ARO Grant W911NF-06-1-0076. Preliminary versions (without detailed development or analysis) of this work have been presented at two conferences [1], [2].

E. Fox is with the Department of Statistical Science, Duke University, Durham, NC, 27708 USA (e-mail: fox@stat.duke.edu).

E. B. Sudderth is with the Department of Computer Science, Brown University, Providence, RI 02912 USA (e-mail: sudderth@cs.brown.edu).

M. I. Jordan is with the Department of Electrical Engineering and Computer Science and Department of Statistics, University of California, Berkeley, CA 94720 USA (e-mail: jordan@eecs.berkeley.edu).

A. S. Willsky is with the Department of Electrical Engineering and Computer Science, MIT, Cambridge, MA 02139 USA (e-mail: willsky@mit.edu).

This paper has supplementary downloadable multimedia material available at http://ieeexplore.ieee.org provided by the authors. This includes detailed derivations of posterior distributions and the filtering and message passing algorithms.

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TSP.2010.2102756