concentration function, and B_0 is a fixed continuous measure on $(\Omega, \mathcal{B})$, called the *base measure*. When c is a constant, we call it the *concentration parameter*. Assume $B_0(\Omega) < \infty$, then for any infinitestimal set $d\omega \in \mathcal{B}$, B is a beta process if

$$B(d\omega) \sim Beta(cB_0(d\omega), c(1 - B_0(d\omega)). \tag{3}$$

Every Lévy process is characterized by its *Lévy measure* or *rate measure*. The Lévy measure of the beta process $BP(c, B_0)$ is given by

$$\nu(d\omega, dp) = c(\omega)p^{-1}(1-p)^{c(\omega)-1}dpB_0(d\omega). \tag{4}$$

To draw B from the beta process distribution, we first draw a set of points $(\omega_i, p_i) \in \Omega \times [0, 1]$ from a Poisson process with rate measure ν, where Ω represents a space of atoms and $[0, 1]$ is the space of weights associated with these atoms. Then we define

$$B = \sum_{i=1}^{\infty} p_i \delta_{\omega_i}. \tag{5}$$

This discrete random measure is such that for any measurable set $S \in \Omega$, we have $B(S) = \sum_{i:\omega_i \in S} p_i$.

The beta process provides an infinite collection of probabilities that can be used to parametize the Bernoulli process. Let $B = \sum_{k=1}^{\infty} p_k \delta_{\omega_k}$ be a draw from a beta process. Let the row vector q_i be infinite and binary with the k^{th} value, q_{ik}, generated by

$$q_{ik} \sim Bernoulli(p_k). \tag{6}$$

Then the new measure $X_i = \sum_{k=1}^{\infty} q_{ik}\delta_{\omega_k}$ is a draw from the Bernoulli process $BeP(B)$.

We can link the beta process $B \sim BP(c, B_0)$ and N Bernoulli process iid draws, $X_i \sim BeP(B)$ for $n \in \{1, \dots, N\}$, to generate a random feature matrix Z. To do so, we reorder the atoms in the beta process so that the first K are locations where some Bernoulli process in $\{X_i\}_{n=1}^{N}$ has a non-zero point mass. We further assume that c is a constant and B_0 is continuous with finite total mass $B_0(\Omega) = \gamma$. It turns out that the distribution of the matrix Z, with the atoms in the beta process draw integrated out, is the same as the distribution in (1). In other words, the beta process is the de Finetti mixing distribution underlying the Indian buffet process [23]. We refer to this overall procedure as the beta-Bernoulli process, denoted by $Z \sim BP\text{-}BeP(N, \gamma, c)$. Figure 2 illustrates draws from $BP\text{-}BeP(N, \gamma, c)$ for $N = 20$ and for several values of γ and c [23].

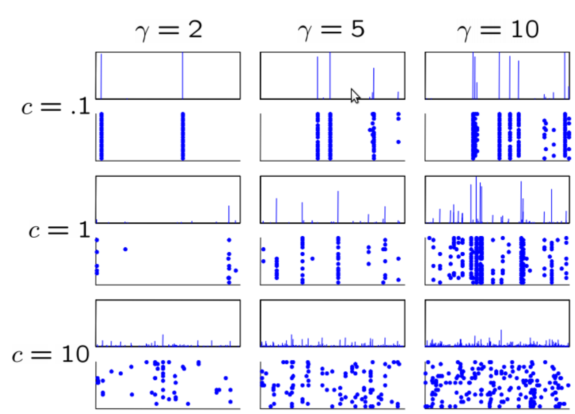

Figure 2: Draws from a beta process with several values of concentrationtion c and uniform base measure with mass γ For each draw, 20 samples are shown from the corresponding Bernoulli process, one per line