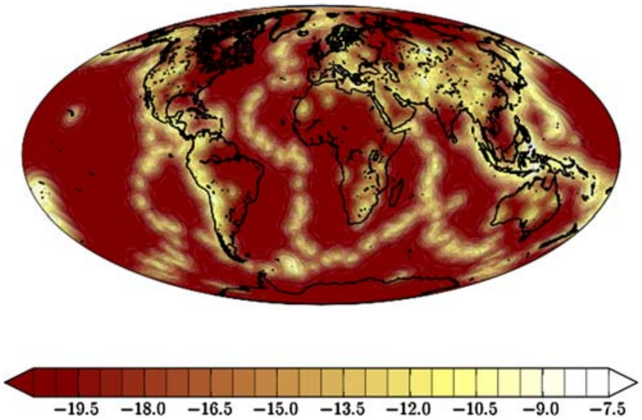

Figure 4. Event location log density. The color version of this figure is available only in the electronic edition.

$$P_{\theta,d}(e_d^i) = \frac{1}{D}. \quad (3)$$

Event Magnitude. The Gutenberg–Richter law (Gutenberg and Richter, 1954) posits that the number of events with magnitude m or more is 10 times the number of events with magnitude $m+1$ or more. In terms of the event magnitude probability of an arbitrary event: $P_{\theta,m}(e_m^i \geq m) = 10 \cdot P_{\theta,m}(e_m^i \geq m+1)$. We represent this prior knowledge in our model with an exponential distribution defined for $e_m^i \geq 2$ with rate $\lambda_m = \log(10)$:

$$P_{\theta,m}(e_m^i) = \lambda_m \exp[-\lambda_m(e_m^i - 2)]. \quad (4)$$

Note that because we assume a minimum magnitude of 2, the exponential distribution is shifted forward by this amount. Although man-made events may not follow the Gutenberg–Richter law, their frequency is too low to affect the overall distribution significantly.

Overall Event Prior. Under the assumption that the event location, depth, time, and magnitude are independent of each other, the probability of a single event, e_i, is given by the product of the component terms:

$$P_\theta(e^i) = P_{\theta,l}(e_l^i)P_{\theta,d}(e_d^i)P_{\theta,t}(e_t^i)P_{\theta,m}(e_m^i).$$

Substituting from equations (2), (3), and (4), we get

$$P_\theta(e^i) = P_{\theta,l}(e_l^i)\frac{1}{D}\frac{1}{T}\lambda_m \exp[-\lambda_m(e_m^i - 2)]. \quad (5)$$

In our model, all the events are exchangeable and are generated independently, thus,

$$P_\theta(e) = P_{\theta,n}(|e|) \cdot |e|! \cdot \prod_{i=1}^{|e|} P_\theta(e^i).$$

(Note that we are overloading $P_\theta(\cdot)$ to refer to the distribution over the set of events as well as the distribution of a single event.) Substituting from equations (1) and (5),

$$P_\theta(e) = \exp(-\lambda_e T)\prod_{i=1}^{|e|} P_{\theta,l}(e_l^i)\frac{1}{D}\lambda_e\lambda_m \exp[-\lambda_m(e_m^i - 2)]. \quad (6)$$

If we define the following single-event quantity, which is independent of the arbitrary interval T,

$$\hat{P}_\theta(e^i) = P_{\theta,l}(e_l^i)\frac{1}{D}\lambda_e\lambda_m \exp[-\lambda_m(e_m^i - 2)].$$

then we can simplify equation (6) to

$$P_\theta(e) = \exp(-\lambda_e T)\prod_{i=1}^{|e|} \hat{P}_\theta(e^i). \quad (7)$$

As noted in the Future Work section, the time-homogeneity and independence assumptions are violated by aftershock phenomena.

True Arrivals

An event can generate at most one true arrival of each phase type at a station. Whether or not the arrival is generated depends on the detection probability.

Detection Probability. The probability that an event i's jth phase, $1 \leq j \leq J$, is detected by a station k, $1 \leq k \leq K$, depends on the phase, the station, and the event's magnitude, depth, and distance to the station (e_m^i, e_d^i, and Δ_{ik}). As noted in the Future Work section, such a model ignores source location and path effects as well as anisotropic radiation patterns. Let $P_{\phi,d}^{jk}(e_i)$ be the probability of this detection. The phase- and station-specific detection distributions, $P_{\phi,d}^{jk}(\cdot)$, were obtained using logistic regression models, which are a standard tool for modeling true/false random variables. In such models, a weighted linear expression is formed from the inputs (and possibly additional features computed from the inputs), and the probability that the output is true; in this case, the detection of the phase is given by applying a soft threshold logistic function to the value of the weighted linear expression. The net effect is that the log odds of detection are a linear function of the input features:

$$\log\left(\frac{P_{\phi,d}^{jk}(e^i)}{1 - P_{\phi,d}^{jk}(e^i)}\right) = \sum_{w \in \mathcal{F}_d} \mu_d^{wjk} \cdot w(e_m^i, e_d^i, \Delta_{ik}),$$

where $\mathcal{F}_d$ is a set of feature functions such that for each $w \in \mathcal{F}_d$, w: $\mathbb{R}^3 \rightarrow \mathbb{R}$. Also, μ_d^{wjk} is the weight for the feature w. The complete set of features is defined in Table 2. Because the event magnitude is one of the features, another way of thinking about the previous equation is that a unit increase