

$\bar{f} = 1 - f$  of its current value  $f$ :

$$(15) \quad \begin{aligned} f_{ik} &\sim \rho(\bar{f}|f)\delta(f_{ik}, \bar{f}) + (1 - \rho(\bar{f}|f))\delta(f_{ik}, f), \\ \rho(\bar{f}|f) &= \min \left\{ \frac{p(f_{ik} = \bar{f} | \mathbf{F}^{-ik}, \mathbf{y}_{1:T_i}^{(i)}, \boldsymbol{\eta}^{(i)}, \theta_{1:K_+^{-i}}, c)}{p(f_{ik} = f | \mathbf{F}^{-ik}, \mathbf{y}_{1:T_i}^{(i)}, \boldsymbol{\eta}^{(i)}, \theta_{1:K_+^{-i}}, c)}, 1 \right\}. \end{aligned}$$

To compute likelihoods  $p(\mathbf{y}_{1:T_i}^{(i)} | \mathbf{f}_i, \boldsymbol{\eta}^{(i)}, \boldsymbol{\theta})$ , we combine  $\mathbf{f}_i$  and  $\boldsymbol{\eta}^{(i)}$  to construct the transition distributions  $\pi_j^{(i)}$  as in equation (10), and marginalize over the possible latent state sequences by applying a forward–backward message passing algorithm for AR-HMMs [see Supplement C.2 of Fox et al. (2014)]. In each sampler iteration, we apply these proposals sequentially to each entry of the feature matrix  $\mathbf{F}$ , visiting each entry one at a time and retaining any accepted proposals to be used as the fixed  $\mathbf{F}^{-ik}$  for subsequent proposals.

6.3. *Sampling state sequences  $\mathbf{z}$ .* For each sequence  $i$  contained in  $\mathbf{z}$ , we block sample  $\mathbf{z}_{1:T_i}^{(i)}$  in one coherent move. This is possible because  $\mathbf{f}_i$  defines a finite AR-HMM for each sequence, enabling dynamic programming with auxiliary variables  $\boldsymbol{\pi}^{(i)}, \boldsymbol{\theta}$ . We compute backward messages  $m_{t+1,t}(z_t^{(i)}) \propto p(\mathbf{y}_{t+1:T_i}^{(i)} | z_t^{(i)}, \tilde{\mathbf{y}}_t^{(i)}, \boldsymbol{\pi}^{(i)}, \boldsymbol{\theta})$ , and recursively sample each  $z_t^{(i)}$ :

$$(16) \quad z_t^{(i)} | z_{t-1}^{(i)}, \mathbf{y}_{1:T_i}^{(i)}, \boldsymbol{\pi}^{(i)}, \boldsymbol{\theta} \sim \pi_{z_{t-1}^{(i)}}^{(i)}(z_t^{(i)}) \mathcal{N}(\mathbf{y}_t^{(i)}; \mathbf{A}_{z_t^{(i)}} \tilde{\mathbf{y}}_t^{(i)}, \Sigma_{z_t^{(i)}}) m_{t+1,t}(z_t^{(i)}).$$

Supplement Algorithm D.3 of Fox et al. (2014) explains backward-filtering, forward-sampling in detail.

6.4. *Sampling auxiliary parameters:  $\boldsymbol{\theta}$  and  $\boldsymbol{\eta}$ .* Given fixed features  $\mathbf{F}$  and state sequences  $\mathbf{z}$ , the posterior over auxiliary parameters factorizes neatly:

$$(17) \quad p(\boldsymbol{\theta}, \boldsymbol{\eta} | \mathbf{F}, \mathbf{z}, \mathbf{y}) = \prod_{k=1}^{K_+} p(\theta_k | \{\mathbf{y}_t^{(i)} : z_t^{(i)} = k\}) \prod_{i=1}^N p(\boldsymbol{\eta}^{(i)} | \mathbf{z}^{(i)}, \mathbf{f}_i).$$

We can thus sample each  $\theta_k$  and  $\boldsymbol{\eta}^{(i)}$  independently, as outlined below.

*Transition weights  $\boldsymbol{\eta}^{(i)}$ .* Given state sequence  $\mathbf{z}^{(i)}$  and features  $\mathbf{f}_i$ , sequence  $i$ 's Markov transition weights  $\boldsymbol{\eta}^{(i)}$  have posterior distribution

$$(18) \quad p(\boldsymbol{\eta}_{jk}^{(i)} | \mathbf{z}^{(i)}, f_{ij} = 1, f_{ik} = 1) \propto \frac{(\boldsymbol{\eta}_{jk}^{(i)})^{n_{jk}^{(i)} + \gamma + \kappa \delta(j,k)} - 1 e^{-\boldsymbol{\eta}_{jk}^{(i)}}}{[\sum_{k': f_{ik'}=1} \boldsymbol{\eta}_{jk'}^{(i)}]^{n_j^{(i)}}},$$

where  $n_{jk}^{(i)}$  counts the transitions from state  $j$  to  $k$  in  $z_{1:T_i}^{(i)}$ , and  $n_j^{(i)} = \sum_k n_{jk}^{(i)}$  counts all transitions out of state  $j$ .