

equation (12) is useful in reminding the reader that the indices of  $\tilde{\pi}_j^{(i)}$  defined by equation (11) are not over 1 to  $K_i$ , but rather over the  $K_i$  values of  $k$  such that  $f_{ik} = 1$ . Additionally, this notation is useful for concise representations of the posterior distribution.

We construct the model using the unnormalized transition weights  $\eta^{(i)}$  instead of just the proper distributions  $\pi^{(i)}$  so that we may consider adding or removing states when sampling from the nonparametric posterior. Working with  $\eta^{(i)}$  here simplifies expressions, since we need not worry about the normalization constraint required with  $\pi^{(i)}$ .

4.3. *VAR parameters.* To complete the Bayesian model specification, a conjugate matrix-normal inverse-Wishart (MNIW) prior [cf., West and Harrison (1997)] is placed on the shared collection of dynamic parameters  $\theta_k = \{\mathbf{A}_k, \Sigma_k\}$ . Specifically, this prior is comprised of an inverse Wishart prior on  $\Sigma_k$  and (conditionally) a matrix normal prior on  $\mathbf{A}_k$ :

$$(13) \quad \begin{aligned} \Sigma_k | n_0, S_0 &\sim \text{IW}(n_0, S_0), \\ \mathbf{A}_k | \Sigma_k, M, L &\sim \mathcal{MN}(\mathbf{A}_k; M, \Sigma_k, L), \end{aligned}$$

with  $n_0$  the degrees of freedom,  $S_0$  the scale matrix,  $M$  the mean dynamic matrix, and  $L$  a matrix that together with  $\Sigma_k$  defines the covariance of  $A_k$ . This prior defines the base measure  $B_0$  up to the total mass parameter  $\alpha$ , which has to be separately assigned (see Section 6.5). The MNIW density function is provided in the supplemental article [Fox et al. (2014)].

**5. Model overview.** Our beta-process-based featural model couples the dynamic behaviors exhibited by different time series. We term the resulting model the *BP-autoregressive-HMM* (BP-AR-HMM). Figure 3 provides a graphical model representation. Considering the *feature space* (i.e., set of autoregressive parameters) and the *temporal dynamics* (i.e., set of transition distributions) as separate dimensions, one can think of the BP-AR-HMM as a spatio-temporal process comprised of a (continuous) beta process in space and discrete-time Markovian dynamics in time. The overall model specification is summarized as follows:

- (1) Draw beta process realization  $B \sim \text{BP}(c, B_0)$ :

$$B = \sum_{k=1}^{\infty} \omega_k \theta_k \quad \text{where } \theta_k = \{\mathbf{A}_k, \Sigma_k\}.$$

- (2) For each sequence  $i$  from 1 to  $N$ :
  - (a) Draw feature vector  $\mathbf{f}_i | B \sim \text{BeP}(B)$ .
  - (b) Draw feature-constrained transition distributions

$$\pi_j^{(i)} | \mathbf{f}_i \sim \text{Dir}([\dots, \gamma + \delta(j, k)\kappa, \dots] \odot \mathbf{f}_i).$$