Pairwise Loop Series Expansion

$$Z(q) = 1 + \sum_{\emptyset \neq F \subseteq E} \mathbb{E}_{\tilde{q}}\left[\prod_{(s,t) \in F} \beta_{st}(X_s - \tau_s)(X_t - \tau_t) \right]$$

$$= 1 + \sum_{\emptyset \neq F \subseteq E} \beta_F \prod_{s \in V} \mathbb{E}_{q_s}\left[(X_s - \tau_s)^{d_s(F)} \right]$$

$$\beta_F := \prod_{(s,t) \in F} \beta_{st}$$

$d_s(F)$ ⟶ degree of node s in *subgraph* induced by F

- Depends on *central pseudo-moments* corresponding to loopy BP fixed point: $\mathbb{E}_{q_s}\left[(X_s - \tau_s)^{d_s(F)} \right]$
- Only *generalized loops* are non-zero: $\mathbb{E}_{q_s}[X_s - \tau_s] = 0$