Binary Markov Random Fields

Boltzmann Machines, Ising Models, …

$$p(x) = \frac{1}{Z} \prod_{s \in V} \psi_s(x_s) \prod_{(s,t) \in E} \psi_{st}(x_s, x_t)$$

- Nodes associated with binary variables: $x_s \in \{0, 1\}$
- Parameterize *pseudo-marginal* distributions via moments:

$$\tau_s := q_s(X_s = 1)$$

$$\tau_t := q_t(X_t = 1)$$

$$\tau_{st} := q_{st}(X_s = 1, X_t = 1)$$

$$q_{st}(x_s, x_t) = \begin{array}{c} \\ x_s \\ 0 \\ 1 \end{array} \begin{array}{c} \begin{array}{cc} 0 & 1 \end{array} \\ \begin{bmatrix} 1 - \tau_s - \tau_t + \tau_{st} & \tau_t - \tau_{st} \\ \tau_s - \tau_{st} & \tau_{st} \end{bmatrix} \end{array} x_t$$