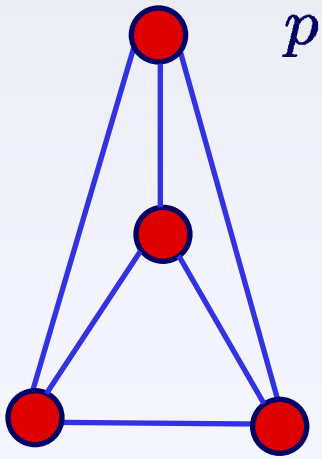
Bethe Approximations & Loopy BP



$$p(x) = \frac{1}{Z} \prod_{s \in V} \psi_s(x_s) \prod_{(s,t) \in E} \psi_{st}(x_s, x_t)$$

$$= \frac{1}{Z(q)} \prod_{s \in V} q_s(x_s) \prod_{(s,t) \in E} \frac{q_{st}(x_s, x_t)}{q_s(x_s) q_t(x_t)}$$

Pseudo-Marginals

- Fixed points of *loopy BP* also correspond to reparameterizations of $p(x)$ *(Wainwright et. al. 2001)*

Bethe variational approximation parameterized by pseudo-marginals which may be globally inconsistent

$$\log Z_\beta = \max_{q=\{q_s, q_{st}\}} H_\beta(q) + \sum_x q(x) \log \psi(x)$$

subject to $\sum_{x_s} q_{st}(x_s, x_t) = q_t(x_t) \qquad \sum_{x_s} q_s(x_s) = 1$

$$H_\beta(q) = \sum_{s \in V} H_s(q_s) - \sum_{(s,t) \in E} I_{st}(q_{st})$$

Yedidia, Freeman, & Weiss 2000