in a 25-km-thick ice shell with such a rheology if the grain size is about 1 mm, which may be the case if grain size is controlled by tides (see section 3.3.2).

Here, we have calculated critical ice shell thicknesses for convection assuming a constant thermal conductivity for the ice shell. The critical ice shell thickness for convection in a variable-conductivity shell is larger than in a constant-conductivity shell. This effect can be estimated by equating the equivalent heat flow F_{conv} across a shell with variable conductivity to F_{conv} with a constant conductivity (*McKinnon,* 1999, 2006; *Tobie et al.,* 2003; *Barr and Pappalardo,* 2005) (see also equations (2) and (3)).

$$\frac{D_{true}}{D_{cr}} = \frac{a_c}{k_c \Delta T} \ln\left(\frac{T_m}{T_s}\right) \tag{30}$$

where D_{true} is the actual critical shell thickness with variable conductivity taken into account, D_{cr} is the value obtained assuming a constant conductivity of k_c (here, 3.3 W m^{-1} K^{-1}). For T_s = 100 and T_m = 260 K, $D_{true}/D_{cr} \sim 1.17$.

Finally, we note that although it is beyond the scope of this chapter, it is possible that the microphysical processes that accommodate the first ~10% strain during the onset of convection are entirely different than those that govern well-developed convection (see, e.g., *Birger,* 1998, 2000). Measurements of ice behavior during transient or primary creep (*Glen,* 1955) may be relevant to the question of the onset of convection in addition to flow laws for steady-state creep (*Solomatov and Barr,* 2007).

The results of recent efforts to refine the range of critical ice shell thickness for convection, which predict critical thicknesses from 10 km to a few tens of kilometers, generally agree with the original estimates derived by *Reynolds and Cassen* (1979): $D_{cr} \sim 30$ km. Although the *value* of critical ice shell thickness may not have changed much in 30 years, the relationship between ice rheology, the critical shell thickness for convection, and ice grain size has been clarified.

4.2. Behavior of the Icy Shell Close to the Critical Rayleigh Number

4.2.1. Starting convection. In the previous section, we described the results of recent studies attempting to narrow the range of conditions where convection is *possible* in Europa's ice shell. In a mathematically idealized scenario, an unperturbed and heated ice shell will sit quiescently unless temperature fluctuations drive flow and trigger convection. Since the earliest work on convective stability, it has been known that the critical Rayleigh number depends on the shape (e.g., *Turcotte and Schubert,* 2002) and amplitude of temperature perturbation within the fluid layer (see, e.g., *Chandrasekhar,* 1961; *Stengel et al.,* 1982, for discussion).

A key open question is whether tidal dissipation can trigger convection in Europa's icy shell. Because the Maxwell time of warm ice near the base of Europa's shell may be close to Europa's orbital period, a purely conductive ice shell may be heated largely at its base. Recent numerical work suggests that maximally effective perturbations for starting convection are concentrated at the base of the fluid layer and have wavelength $\lambda_{cr} \sim 2(n + 3)\theta^{-1}D$ (*Solomatov and Barr,* 2006, 2007). In the absence of temperature fluctuations, zones of weakness, or other means of localizing tidal dissipation, tidal heating in a conductive ice shell is essentially constant over the horizontal scale of convective cells because the r.m.s. strain rate varies by only a factor of ~2 between the equator to pole. One could envision the temperature perturbation due to tidal dissipation as a smoothly varying harmonic function with a wavelength $\lambda_{tidal} \sim R_{Europa}/4$ or so (see Fig. 1 of *Ojakangas and Stevenson,* 1989). If tidal heating is the sole cause of the density differences necessary to trigger convection, tidal dissipation must generate temperature perturbations on horizontal length scales $\lambda \sim \lambda_{cr}$ to trigger convection (*Barr et al.,* 2004). If $\lambda_{tidal} \gg \lambda_{cr}$, the critical Rayleigh number would increase substantially, perhaps by a factor of 100 or more, because perturbations with such long wavelengths are inefficient at triggering convection. This suggests that tidal dissipation, as envisioned by *Ojakangas and Stevenson* (1989), may not be able to trigger convection in a *purely* conductive icy shell.

Other types of temperature fluctuations, e.g., bursts of heat released close to the surface of the icy shell from large impacts, are essentially useless in triggering convection because they diffuse away too quickly to warm the surrounding ice enough to permit it to flow. Compositional variations present in a realistic icy shell may be able to provide the necessary density contrasts to trigger convection (e.g., *Pappalardo and Barr,* 2004). Understanding how convection begins in Europa's icy shell will require characterization of the types and locations of temperature fluctuations naturally present in the icy shell.

4.2.2. Stopping convection. If the Rayleigh number of Europa's icy shell drops below the value where convection can be maintained, convection will cease. The Ra of the ice shell may change, for example, due to perturbations in the basal heat flux (*Mitri and Showman,* 2005), or due to an increase in ice grain size over time (*Barr and McKinnon,* 2007). Gray arrows in Fig. 1c describe the path in Ra-Nu space taken by an ice shell where convection is stopping. As the Rayleigh number is decreased, Nu decreases until $Ra = Ra^*_{cr}$, the lowest value of Ra where convection is possible. For Newtonian rheologies in the stagnant lid regime ($\theta > 8$), the value of $Ra^*_{cr} \sim \frac{1}{2}Ra_{cr,1}$ (see Table 2). In the vicinity of this point, $(Nu - Nu_{cr}) \propto (Ra - Ra_{cr})^{1/2}$, and at $Ra = Ra^*_{cr}$, $Nu \approx 1.1$ to 1.3, and convective motion is confined to a very thin layer at the base of the ice shell. Values of Ra^*_{cr} and $Nu(Ra^*_{cr})$ for a range of parameters appropriate for a volume diffusion rheology and range of θ appropriate for Europa's icy shell are summarized in Table 2 (see also Table 1 of *Solomatov and Barr*, 2007). When convection stops, very low values of (Nu – 1) can be achieved, and the minimum value scales with θ^{-1} (*Solomatov and Barr,* 2007).

4.2.3. Conductive-convective switching. When convection starts in an icy shell, it results in a reorganization of