when dr/dt = 0, which gives the Zener limiting grain size (cf. *Durand et al.,* 2006)

$$r_z = \left(\frac{3\alpha_G}{\pi r_x f N_x}\right)^{1/2} \tag{20}$$

where the numerical factor of 3 indicates that the particles reside on grain boundaries, and f ~ 0.25 is estimated by counting the number of particles residing on grain boundaries in SEM images of the GRIP ice core (*Barnes et al.,* 2002; *Weiss et al.,* 2002). If we suppose that the ice shell of Europa is loaded with microscopic silicate particles of density 3000 kg m^{-3} and radii of 10 µm, up to a total volume percentage of 4%, the Zener limiting grain size is

$$r_z = 0.1\ \text{mm}\left(\frac{r_x}{10\ \mu\text{m}}\right)\left(\frac{0.04}{\phi}\right)^{1/2} \tag{21}$$

where ϕ is the volume fraction of silicate in Europa's icy shell and $N_x \sim \phi/\left(\frac{4}{3}\pi r_x^3\right)$. The upper limit on ice grain size derived from the Zener pinning model is inversely proportional to impurity content — fewer impurities mean larger grain sizes. We note that *Kirk and Stevenson* (1987) constructed a similar argument to estimate grain sizes in the ice mantle of Ganymede: Their estimate has a similar dependence on radii of the silicate particles but assumes randomly distributed particles and gives $r_z \sim \phi^{-1}$. For Europa, we can put a plausible estimate on the upper limit of silicate content based on the density of the icy shell — if the shell density exceeds 1000 kg m^{-3} (or more for a salty ocean), it will be gravitationally unstable atop the ocean.

Observations of grain sizes between 0.5 and 1 mm in impurity-laden sections of terrestrial ice cores, coupled with our upper limit on ice grain size based on a Zener pinning model, suggest that grain sizes in Europa's icy shell may hover around the 0.1–1-mm range (*McKinnon,* 1999; *Barr and McKinnon,* 2007). At present, terrestrial observations of grain growth and impurity distribution within polycrystalline ice are limited to temperatures between 235 to 273 K. Therefore, knowledge of grain size and processes controlling grain size gained through study of terrestrial cores may be most applicable to the warm and convecting interior ice shell (*Barr and McKinnon,* 2007). Grain sizes closer to the surface of Europa may be controlled by other, non-thermally activated processes such as cyclical tidal deformation (*McKinnon,* 1999).

3.3.2. Dynamic recrystallization. Observations of relatively impurity-free sections of terrestrial ice cores reveal that ice grain sizes are constant as a function of depth (equivalently, time) within sections of the core that have experienced significant strain (*Thorsteinsson et al.,* 1997; *De La Chapelle et al.,* 1998). If ice grains can grow unimpeded, one would expect grain size in the ice sheet to increase as a function of depth and time. This suggests that deformation acts to decrease grain size, thereby competing with natural grain growth and allowing a roughly constant steady-state grain size to be maintained over time. It has been suggested that the accumulated strain due to vertical layer compaction results in grain size reduction due to a process called dynamic recrystallization (*Thorsteinsson et al.,* 1997; *De La Chapelle et al.,* 1998). In dynamic recrystallization, the grain size in a deforming material is controlled by a balance between grain growth and the formation of new grains (nucleation) by a process called subgrain rotation (*Shimizu,* 1998; *DeBresser et al.,* 1998). Subgrain rotation can only occur if deformation is occurring in the material, leading to a threshold strain at which grain sizes in a deforming material achieve their steady-state recrystallized values. For temperature and strain rate conditions appropriate for the GRIP ice core, T ~ 240 K and $\dot{\varepsilon} \sim 10^{-12}\ s^{-1}$, the threshold strain is about 25% (*Thorsteinsson et al.,* 1997). A model of this process has been applied to estimate grain sizes within actively deforming regions of ice shells by *Barr and McKinnon* (2007), who find that in the well-mixed, warm convective interior of an already convecting ice shell, grain sizes will evolve to a steady-state value that depends on the applied stress (*Derby,* 1991; *Shimizu,* 1998; *DeBresser et al.,* 1998; *Barr and McKinnon,* 2007)

$$d_{recrys} = Kb\left(\frac{\sigma}{\mu}\right)^{-m} \tag{22}$$

where K = 1–100 is a grouped material parameter, µ is the ice shear modulus, and m = 1.25. *Barr and McKinnon* (2007) suggest that in the absence of impurities, recrystallized grain sizes in convecting ice shells will be large, d_{recrys} ~ 30–80 mm, which may lead to highly viscous ice and a gradual shut-down of convection as the grains achieve their recrystallized value. The implication is that without impurities to limit grain growth, ice shells may convect sluggishly, and may be limited to a small number of convective overturns before transitioning to a conductive state.

On Europa, however, tidal flexing of the icy shell itself may control the ice grain size. *McKinnon* (1999) hypothesized that if cyclical straining in the presence of convection of Europa's icy shell has the same effect as the continuous strain on terrestrial ice cores (i.e., driving dynamic recrystallization), the grain size in the icy shell would decrease as $d \propto \dot{\varepsilon}^{-1/2}$. The grain size controlled in this manner would have a *maximum* of 1 mm at the warm base of the icy shell. Thus, cyclical tidal flexing may prevent grains in Europa's icy shell from growing to the large values predicted by continuous-deformation dynamic recrystallization models (*Barr and McKinnon,* 2007), exempting Europa's icy shell from being choked while convecting.

4. ICY SHELL CONVECTION

In section 2, we described two possible modes of removing tidal heat generated in Europa's icy shell: conduction and convection. But how do we decide whether convection can happen? Until recently, knowledge of how convection starts and stops in realistic planetary mantles was relatively limited because terrestrial planet mantles are commonly assumed to convect throughout most of their geological history. Although many of the techniques developed for studying terrestrial planet mantle convection apply to Europa, the heat flow history in Europa's icy shell sets it apart from