2.2. A Tidally Heated Conductive Ice Shell

State-of-the-art models of tidal heating in Europa's icy shell calculate the dissipation occurring in the shell due to its cyclical diurnal tidal flexure by modeling the icy shell as a Maxwell viscoelastic solid. For a general discussion of the Maxwell model, see *Ojakangas and Stevenson* (1989) and *Turcotte and Schubert* (2002). The energy dissipated in a Maxwell solid is maximized when the period of the forcing $T \sim \tau_M = \eta/\mu$, where τ_M is the Maxwell time, η is the viscosity, and μ is the shear modulus of the material. By coincidence (or orbital and geophysical "tuning" in the Jupiter system), the orbital period of Europa (3.5 days) is very close to the Maxwell time for warm ice I ($\mu \sim 3.5 \times 10^9$ Pa and $\eta \sim 10^{15}$ Pa s, which gives $\tau_M \approx 3$ days). A warm, internally heated ice shell on Europa ice shell may be close to a maximally dissipative state.

In a Maxwell viscoelastic solid, the volumetric dissipation rate (q) is proportional to viscosity, $q \sim \eta(T)\dot{\varepsilon}^2$ at high temperatures, and inversely proportional to viscosity, $q \sim \mu^2\dot{\varepsilon}^2/[\omega^2\eta(T)]$, at low temperatures. This implies a strongly temperature-dependent dissipation rate with peak dissipation occurring between ~220 and 270 K depending on the ice grain size (see below). A hot ice shell can therefore be more dissipative than a cold ice shell, and the dissipation should depend on the shell thickness (*Cassen et al.,* 1980). This will affect the existence of equilibria between tidal dissipation and heat transfer.

To date, the most physically realistic model of a tidally heated conductive europan ice shell was proposed by *Ojakangas and Stevenson* (1989), who related the thickness of the ice shell, D, to the heat flux from the deeper interior F_{core}

$$D \approx \frac{\ln(T_b/T_s)}{\left[\left(\frac{2}{a_c}\right)\int_0^{T_b} \frac{q(T)dT}{T} + \left(\frac{F_{core}}{a_c}\right)^2\right]^{1/2}} \tag{4}$$

where $a_c = 621$ W m^{-1} (*Petrenko and Whitworth,* 1999) (see also equation (3)). Physically, this equation describes a conductive equilibrium in an internally heated ice shell with basal heat flux F_{core} and temperature-dependent thermal conductivity. If the ice is modeled as a Maxwell viscoelastic solid, the tidal dissipation rate is (*Ojakangas and Stevenson,* 1989)

$$q = \frac{2\mu\langle\dot{\varepsilon}_{ij}^2\rangle}{\omega}\left[\frac{\omega\tau_M}{1 + (\omega\tau_M)^2}\right] \tag{5}$$

where $\langle\dot{\varepsilon}_{ij}^2\rangle$ is the time-average of the square of the second invariant of the strain rate tensor, $\tau_M = \eta(T)/\mu$ is the temperature-dependent Maxwell time, and $\omega = n$. The dissipation rate maximizes when $\omega \sim \tau_M^{-1}$, corresponding to temperatures of ~220–270 K for ice viscosity of ~10^{13}–10^{15} Pa s, is close to plausible viscosities for warm ice (see discussion in section 3). Equation (5) suggests that tidal dissipation occurs in the warmest ice, and that tidal dissipation in the cold, stiff portions of the icy shell is negligible. Assuming that the ice shell has horizontally uniform material properties, the quantity $\langle\dot{\varepsilon}_{ij}^2\rangle$ is a low-degree spherical harmonic function that varies by a factor of 2 between the minimum at the subjovian point and the maximum at the poles (see Fig. 1 of *Ojakangas and Stevenson,* 1989). The surface temperature on Europa also varies significantly, ranging from $T_s \sim 52$ K at the poles to $T_s \sim 110$ K at the equator. Integration of equation (4) with the tidal heat source (equation (5)) and including the spatially varying surface temperature gives the equilibrium ice shell thickness ranging from ~15 to 30 km as a function of location on Europa assuming radiogenic heating from the rocky mantle is 10 mW m^{-2} (*Ojakangas and Stevenson,* 1989).

Since its development, spacecraft data and advances in our understanding of the behavior of a floating ice shell have questioned the applicability of the *Ojakangas and Stevenson* (1989) model. Galileo images of pits, chaos, and uplifts on Europa's surface suggesting a convecting icy shell, and the suggestion that a shell $D \geq 30$ km thick could convect (see section 4), imply that the icy shell could be much thicker and still carry the tidal heat flux. The *Ojakangas and Stevenson* (1989) model, however, is an extremely valuable starting point for study of more complicated tidal/convective systems (e.g., *Tobie et al.,* 2003) because it demonstrates that tidal dissipation in Europa's shell is strongly rheology-dependent, and suggests that variations in tidal dissipation and surface temperature may lead to variations in the activity within the shell and potential for resurfacing.

2.3. Convection

Europa's ice shell may also transport heat by solid-state convection. The density of water ice, like most solids, decreases as a function of increasing temperature. Therefore, an ice shell cooled from its surface and heated from within and beneath is gravitationally unstable: Warm ice rises, cold ice sinks, which transports thermal energy upward to the base of a conductive "lid" at the surface of the icy shell. When this process is self-sustaining over geologically long timescales, it is called solid-state convection.

Thermally driven convection in a highly viscous fluid with no inertial forces is described by the conservation equations for mass, momentum, and energy (cf. *Schubert et al.,* 2001)

$$\nabla \cdot \vec{v} = 0 \tag{6}$$

$$\nabla p - \rho_o\alpha(T - T_o)g\hat{e}_z = \nabla \cdot [\eta(\nabla\vec{v} + \nabla^T\vec{v})] \tag{7}$$

$$\vec{v} \cdot \nabla T + \frac{\partial T}{\partial t} = \kappa\nabla^2 T + \gamma \tag{8}$$

where $\vec{v}$ is the velocity field, T is temperature, t is time, α