Fig. 2c compares our viscoelastic results with the flattening results, by plotting the maximum stress as a function of the phase shift. The general shapes of the viscoelastic and flattening curves are similar. Though for a given phase shift the flattening model tends to overestimate the maximum stress amplitude by a factor of about 1.5. Alternatively, for a given maximum stress the flattening model predicts a larger phase shift. The results thus show that the viscoelastic and flattening models predict different relationships between the magnitude of the NSR stresses and their location on the surface of the satellite. Either the phase shift or the magnitude of the NSR stresses can match, but not both simultaneously. Note that had we not allowed shear stresses to relax in the synchronously rotating core by using a low value of $\tilde{\mu}_{\text{core}}$ in the NSR Love number calculations, as discussed in Section 7 above, the real parts of the NSR Love numbers would have been similar to the diurnal values (see Figs. 1a and 1b, and Tables 2 and 3), resulting in the viscoelastic stresses being $\sim$35% smaller, and increasing the difference from the flattening model even more.

To compare spatial stress patterns for the two different models of NSR stresses, we need to match up values of Δ in the viscoelastic model with corresponding values of accumulated NSR in the flattening model. The results shown in Figs. 2a and 2b indicate that small values of accumulated NSR correspond to large vales of Δ (i.e. to long NSR periods and/or small viscosities). For every value of accumulated NSR in the flattening model we find the corresponding value of Δ in the viscoelastic model such that the two models give identical phase shifts for the maximum tensile stress. Because of the linear relationship between phase shift and the amount of accumulated NSR evident in Fig. 2b, we expect 1° of NSR to result in a phase lag of 44.5° $(= 45° - (0.5 \times \text{NSR}))$. For the parameters used in these calculations, this corresponds to $\Delta = 56$. The comparison can be seen in Figs. 5c and 5d. The spatial patterns are in good agreement. The amplitudes of the flattening stresses are larger than the amplitudes of the viscoelastic stresses, but only by $\sim$20% rather than the $\sim$50% that might be expected from Fig. 2c. This is because Figs. 5c and 5d consider a phase shift close to 45°, which is where the flattening model has usually been applied in the past. Fig. 2c shows that as the phase shift gets close to 45° the amplitude of the viscoelastic stresses approaches and eventually even exceeds that of the flattening stresses, though both are small at large values of Δ. This is partly due to the buoyancy mode described at the end of Section 8.1, which begins to influence the surface stresses when Δ exceeds $\sim$10 (see Figs. 1a, 1b, and 2a). The flattening stresses reach zero amplitude when the phase shift is 45°, but the viscoelastic stresses maintain an amplitude of at least a few kPa until the phase shift is close to 50°, and vanish only as $\Delta \to \infty$.

10. Summary and future work

We have developed and implemented a method of calculating the tidally induced surface stresses of a radially stratified satellite with a Maxwell viscoelastic shell of arbitrary thickness overlying an inviscid ocean and a silicate core, derived directly from the time-varying gravitational potential experienced by the satellite. All regions of the satellite are compressible and self-gravitating. The formalism could easily be extended to also include viscoelasticity within the silicate core, though we have chosen not to do so here. The results could also readily be extended to find the stress field at any depth within the shell, by using output from the numerical Love number code at subsurface depths.

We have applied this model to radial and librational diurnal tides caused by the eccentricity of the satellite's orbit, and to tides that would be caused by faster than synchronous rotation of a floating shell. In both these cases we assumed the satellite's orbit has zero obliquity, so that the orbital motion is in the satellite's equatorial plane, and that the NSR motion occurs in that same plane.

Viscoelastic effects are incorporated through the use of frequency-dependent, complex-valued Lamé parameters and Love numbers. The inclusion of viscous relaxation has significant implications for the NSR stress environment at the satellite's surface, both reducing the magnitude of stresses due to long period forcings, and inducing a phase shift that translates the NSR stress field in the opposite direction of shell rotation. The importance of these effects depends on the ratio of the NSR period to the viscous relaxation time of the satellite's outer surface, a ratio described here by the parameter Δ. If $\Delta \lesssim 10$, NSR stresses are much larger than diurnal stresses, and are very similar to the elastic limit with a $\sim$0° phase shift. If $\Delta \gtrsim 100$, the NSR stresses will have a phase shift of $\sim$45°, but their amplitude will be smaller than the diurnal stresses. The effects of viscoelasticity on the diurnal stresses are insignificant for any plausible value of outer surface viscosity.

Because Δ affects the phase shift of the stress field, the apparent longitude of formation of a lineament will also depend on Δ. If we accept the possibility that Δ changes through time, this makes it more difficult to use a lineament's apparent longitude of formation as a proxy for its time of formation, even relative to other lineaments, since they may have formed under NSR stress regimes with different phase shifts.

If we think the linear features observed on the surface of an icy satellite are tidally induced or influenced fractures, it must follow that the surface stresses sometimes exceed the strength of the icy lithosphere. This implies that localized stress release due to brittle failure plays a role in defining the surface stress environment (Smith-Konter and Pappalardo, 2008). It would be beneficial to incorporate the formation of brittle fractures and the resulting changes in the stress field into the viscoelastic model (cf. King et al., 1994). However, that modeling is inherently numerical, requiring localized adjustment of the stresses as each crack forms and affects the formation of subsequent fractures in the region. Thus, the stresses of our model are those one would expect to find on the surface of a viscoelastic shell stronger than the greatest calculated stress. In this paper we have applied this model to the stresses experienced by a shell in steady state with a constant rotation rate, but there are many other possible scenarios for reorientation of a decoupled shell that are not well represented by a steady-state solution. A time-variable NSR rate can easily be accommodated using the formalism described here, while calculating the time evolution of stresses due to episodic polar wander will require enhancements to the model.

11. Sharing the model with the community

Other researchers are encouraged to create their own implementation of the model described in this paper. For those who prefer to use, verify, or build upon our implementation of the model, we are providing access to the code under a public license at: http://code.google.com/p/satstress.

We are also providing a web-based interface to the model at: http://icymoons.com/satstress where users may input model parameters, and perform regularly gridded calculations like those used to generate the figures presented in this paper.

An important benefit of hosting the model on the web is that individual model runs can be archived automatically for later reference. For example, Table 5 contains the unique model run IDs of the SatStress calculations that went into making Figs. 3–5. With one of these IDs a user can view all the model inputs and outputs pertaining to the run. They can also use any run ID as the basis of a new model run. This centralized recordkeeping makes it easier to track and compare model inputs and outputs without having