radial displacements of density discontinuities. The $\Delta \approx 300$ mode evident in Figs. 1a and 1b corresponds to the viscoelastic mode usually referred to as C0 in the post-glacial-rebound literature, associated with the relaxation of the Earth's core-mantle boundary (e.g., Peltier, 1985). When the ocean/shell boundary is displaced from an equipotential surface, there is a gravitational (buoyancy) force that acts to restore it. This force is opposed by viscous resistance within the shell. Although this contribution to the viscoelastic Love numbers is interesting from a dynamical viewpoint, its impact on the surface stresses is minimal because the Love numbers play only a secondary role in determining the viscoelastic contributions to the surface stresses. However, it does cause the maximum phase shift to slightly exceed 45° when $\Delta \gg 1$, as can be seen in Fig. 2a for Europa. Reducing the density contrast between the ice and the ocean reduces the buoyancy force, and increases the timescale on which these forces have an effect, pushing both the large dip in the imaginary part of $\tilde{\ell}$, and the increase in phase shift to values greater than 45°, out to larger Δ values.

8.2. *The direct effects of the outer layer's viscosity*

Viscous effects influence the surface stresses most through the Lamé parameters $\tilde{\mu}$ and $\tilde{\lambda}$ of the upper shell, and their impact on the parameters $\tilde{\beta}_1$, $\tilde{\gamma}_1$, $\tilde{\beta}_2$, $\tilde{\gamma}_2$, and $\tilde{\Gamma}$ (Eqs. (32)–(36)). The surface stress components (Eqs. (29)–(31)) are proportional to those last five parameters, and each of those parameters is proportional to $\tilde{\mu}$ of the outer surface. All except $\tilde{\Gamma}$ also depend on $\tilde{\lambda}$, but that dependence is not very strong. Figs. 1c and 1d show the Lamé parameters as functions of Δ. Over the range of Δ considered here, $\tilde{\mu}$ varies enormously (from zero to 3.5×10^9 Pa) but $\tilde{\lambda}$ varies by only ~30% (from ~7 to $\sim 9 \times 10^9$ Pa). This variability, combined with the direct influence of $\tilde{\mu}$ on all of the parameters listed above, implies that the Δ dependence of $\tilde{\mu}$ will strongly and directly impact the surface stresses.

Note from Eq. (24) that $\mathrm{Im}(\tilde{\mu})/\mathrm{Re}(\tilde{\mu}) = \Delta$. Thus when $\Delta \ll 1$ the imaginary part is small relative to the real part, and when $\Delta \gg 1$ it is large (though both the real and imaginary parts vanish as $\Delta \to \infty$). When the real and imaginary parts are of comparable magnitude (i.e. $0.1 < \Delta < 10$), $\tilde{\mu}$ departs significantly from either the fluid or elastic limits. All of these characteristics of $\tilde{\mu}$ get passed through directly to the stress components. The value of Δ at the outer surface thus becomes critical in determining the surface stresses.

For the diurnal tides, assuming a plausible value for the viscosity of cold surface ice of 10^{22} Pa s (Table 1), Δ in the upper layer is roughly 2×10^{-8}, which means the effects of viscoelasticity can be safely ignored. The outer surface viscosity would have to be as small as 2×10^{15} Pa s for Δ to be as large as 0.1, which is roughly when viscous effects start to become important. A viscosity of 2×10^{15} Pa s may be reasonable for the warm lower ice layer, but as described above, the lower layer Δ has only a minimal impact on the surface stresses, no matter its value. The implication is that viscoelastic effects are not likely to have a significant effect on the diurnal tidal stresses.

For the NSR tides (again assuming the outer layer viscosity is 10^{22} Pa s), if the NSR period is between 1.2×10^5 and 1.2×10^7 yr, then Δ is in the range $0.1 < \Delta < 10$ where viscoelastic effects are important and are very sensitive to Δ. If the NSR period is significantly longer than 1.2×10^7 yr, then $\Delta \gg 1$, and the surface stresses are small: they decay away almost as quickly as the forcing can create them.

However, even relatively small NSR stresses can overwhelm the diurnal stresses. The diurnal tides arise because of the orbital eccentricity ϵ, and cause displacements that are ϵ ($= 0.0094$ for Europa) times as large as the NSR displacements. Thus, if the ice layer was elastic, the diurnal stresses would similarly be a factor of ϵ times smaller than the NSR stresses. Viscoelasticity reduces the NSR stress magnitudes as Δ increases (i.e. as the NSR period gets longer). For $\Delta \gg 1$, the parameters $\tilde{\beta}_1$, $\tilde{\gamma}_1$, $\tilde{\beta}_2$, $\tilde{\gamma}_2$, and $\tilde{\Gamma}$ in Eqs. (32)–(36) become approximately inversely proportional to Δ. The elastic case is given by those same expressions, but with $\Delta = 0$. Thus, in order for the amplitudes of the NSR stresses to be reduced to where they are comparable or smaller than the diurnal amplitudes, Δ has to be on the order of $1/\epsilon$ or larger (~100 for Europa). For an outer layer viscosity of 10^{22} Pa s, an NSR period of $\sim 1.2 \times 10^8$ yr or longer is required for diurnal stresses to dominate NSR stresses.

8.3. *Stress patterns and phase shifts*

The value of Δ is critical for determining not only the magnitude of the surface stresses, but also how those stresses are distributed over the surface of the satellite. The smaller the value of Δ, the closer the shell's response is to being elastic, and so the closer the stresses are to being oriented symmetrically about the planet–satellite vector — a state we will refer to as having zero phase shift.

For the diurnal tides, Δ is so small that the resulting stresses are virtually elastic (see above). If the default viscosity values given in Table 1 are altered such that the entire shell has the high viscosity of the surface (10^{22} Pa s), the amplitude of the diurnal component of the stresses changes by less than 0.1%. If the entire low viscosity portion of the shell is replaced with ocean, leaving only an 8 km thick high-viscosity upper shell, the difference in the amplitude of the diurnal component of the stresses is ~3%. Maps of the diurnal stresses at different points in the orbit (i.e. at different values of nt in Eqs. (29)–(31)) are shown in Fig. 3. In these plots, the sub-jovian point at perijove is at latitude (y-axis) 0°, longitude (x-axis) 0°.

For the NSR tides the value of the period is unknown. Both the amplitude and the phase shift of the stresses depend on that period, mostly through the direct dependence on Δ of the outer surface. The parameters $\tilde{\beta}_1$, $\tilde{\gamma}_1$, $\tilde{\beta}_2$, $\tilde{\gamma}_2$, and $\tilde{\Gamma}$ (Eqs. (32)–(36)) are proportional to $\tilde{\mu}$, and $\tilde{\mu}$ depends on Δ through (24). Thus, the right-hand sides of Eqs. (32)–(36) show that when $\Delta \ll 1$ the parameters $\tilde{\beta}_1$, $\tilde{\gamma}_1$, $\tilde{\beta}_2$, $\tilde{\gamma}_2$, and $\tilde{\Gamma}$ are nearly real and are well approximated by their elastic values (since $\tilde{h}$ and $\tilde{\ell}$ are nearly equal to their elastic values).

When $\Delta \gg 1$, those five parameters have small amplitudes, but their imaginary parts are a factor of Δ larger than their real parts (for example, $1/(1 - i\Delta) = (1 - i\Delta)/(1 + \Delta^2)$, has real and imaginary parts of approximately Δ^{-2} and Δ^{-1} respectively, when $\Delta \gg 1$). This means that while $\tau_{\theta\theta}$ is proportional to $\cos(2\phi + 2bt)$ in the elastic limit, it is nearly proportional to $\sin(2\phi + 2bt)$ in the $\Delta \gg 1$ limit (see Eq. (38)). The factors of 2 in $(2\phi + 2bt)$ mean that the spatial pattern of $\tau_{\theta\theta}$ in the $\Delta \gg 1$ limit is displaced from the elastic pattern by 45°. The same is true of the other stress components, $\tau_{\phi\phi}$ and $\tau_{\phi\theta}$.

In general the patterns and amplitudes of the tidally induced surface stresses can be separated into 4 possible regimes. If the orbital eccentricity ϵ is small and the shell is thin, and assuming for the case of Europa that the viscosity of the upper layer of the ice is $\eta_{\mathrm{upper}} = 10^{22}$ Pa s, we find:

(a) $\Delta < 0.1$. The NSR stress is nearly elastic, and so has a phase shift of ~0°. The NSR stress amplitude is a factor of $\sim \epsilon^{-1}$ times larger than the diurnal stress amplitude. For our nominal Europa, this corresponds to an NSR period (P_{nsr}) of less than 1.2×10^5 yr, and tensile NSR stresses of up to ~3.2 MPa.

(b) $0.1 < \Delta < 10$. The NSR stress amplitude is still much larger than that of the diurnal stress, but it varies rapidly through this range of Δ values (for Europa the NSR stress amplitude