algorithms used by geophysicists to compute tides on the Earth, and involves a modified version of the code used by Dahlen (1976) to compute terrestrial tides (see also Wahr et al., 2006; Rappaport et al., 2008). We have modified this code to include complex rheological parameters and complex solution scalars, so that we can accommodate a Maxwell solid rheology. The diurnal and NSR solutions differ solely through the frequency-dependence of the Lamé parameters, as shown in Eqs. (21)–(22). Once we have determined the complex Love numbers, $\tilde{h}$ and $\tilde{\ell}$, they are used as described in Section 5 to find the surface stresses.

In the formulation of the model described above, we assume the coordinate system is attached to the icy shell and so rotates with it. The equations of motion (Eqs. (6)–(8)) depend on the assumption that all displacements are small as seen in this coordinate system. These are reasonable assumptions for the diurnal tides. However, in the case of NSR it is only the icy shell that rotates nonsynchronously; the rocky core likely remains synchronously locked to the satellite's orbital motion (Greenberg and Weidenschilling, 1984). From the perspective of points in the icy shell, which is the region of most interest to this study, the external gravity field changes partly because the shell is rotating through the parent planet's gravity field, and partly because the shell is rotating through the gravity field caused by the underlying core. At points within the rocky core, on the other hand, the gravity field never changes, and so as described in Section 6, the core always responds to the NSR tidal forcing as though it were a fluid.

To obtain an adequate representation of the shape of the core and its effects on the icy shell, we set $\mu \approx 0$ in the rocky core when solving for the NSR tides. This eliminates shear stresses in the core, leading to much larger core tidal displacements. This in turn significantly increases the displacements and shear stresses in the icy shell, since then the gravity field from the core's tidal bulge can be a much larger fraction of the direct gravity field from the parent planet. On Europa for example, setting $\mu \approx 0$ in the core can increase the displacements and shear stresses in the icy shell by up to 70%. This difference in the assumed behavior of the core under the diurnal and NSR forcings is why there are two sets of each of the Love numbers displayed in Figs. 1a and 1b.

## 8. Results

Our formulation of the equations of motion and the resulting stresses describes the effects of viscoelasticity in terms of the parameter $\Delta$, which is proportional to the ratio between the forcing period and the Maxwell relaxation time of the ice. Because the shell we are considering has two viscoelastic layers with different viscosities, it will also have two values of $\Delta$ for a given forcing period. The resulting stresses at the outer surface (Eqs. (29)–(31)) depend on those values in two ways:

(i) they depend on the $\Delta$ values of each ice layer through the Love numbers $\tilde{h}$ and $\tilde{\ell}$; and
(ii) they depend on $\Delta$ of the upper icy layer only, through $\tilde{\beta}_1$, $\tilde{\gamma}_1$, $\tilde{\beta}_2$, $\tilde{\gamma}_2$, and $\tilde{\Gamma}$.

Item (i) represents the effects of viscoelasticity on the surface displacements, whereas (ii) describes how those displacements translate into stresses within a viscoelastic medium (with the properties of the upper ice layer). The dependence (i) is weak so long as there is an underlying ocean and the ice shell is thin, as we are assuming in this application. $\Delta$ does have a large relative effect on the imaginary parts of the Love numbers, but the imaginary parts are only a small fraction of the real parts. The tidal response in this case is mostly determined by the ocean, and so the Love numbers are only weakly dependent on the properties of the ice shell (see Moore and Schubert, 2000; Wu et al., 2001; Wahr et al., 2006; Rappaport et al., 2008). Thus, the primary influence of viscous effects comes through (ii).

### 8.1. Love numbers

The effects of viscoelasticity on the real and imaginary parts of the Love numbers are shown in Figs. 1a and 1b, for values of $\Delta$ (at the outer surface) spanning nine orders of magnitude. Values of $\Delta \ll 1$ indicate a forcing period short enough that the shear stresses do not have time to relax during a forcing cycle, and so the material behaves nearly elastically. Values of $\Delta \gg 1$ imply a long enough forcing period that the stresses have time to almost completely relax, allowing the material to behave almost as an inviscid fluid.

The results shown in these figures are computed using the upper and lower viscosity values $\eta_{\text{upper}}$ and $\eta_{\text{lower}}$ shown in Table 1, and varying the forcing period. The elastic value of $\mu$, given in Table 1, is assumed to be the same in each layer. Thus, the results are computed assuming the values of $\Delta$ in the lower and upper layers are related by a factor of $\Delta_{\text{lower}}/\Delta_{\text{upper}} = \eta_{\text{upper}}/\eta_{\text{lower}} = 10^5$. We would have obtained the same results if we had fixed the forcing period and varied the viscosities of the layers in tandem (i.e. maintaining $\eta_{\text{upper}}/\eta_{\text{lower}} = 10^5$).

Results are shown both for the diurnal tides, where the period is well known but the viscosity is not, and for the NSR tides, where neither the period nor the viscosity is known. The difference between the diurnal and NSR results at any given value of $\Delta$ is because in the NSR case we assume the silicate core behaves as a fluid, and in the diurnal case we assume it behaves elastically. As described in Section 7, this causes the NSR Love numbers to be about 70% larger than the diurnal Love numbers for a fixed value of $\Delta$.

Figs. 1a and 1b show that the real parts of the Love numbers (thick lines, tied to left axes) are one to two orders of magnitude larger than the imaginary parts (thin lines, right axes) no matter what value is assumed for $\Delta$. This is because the icy lithosphere is thin and so has only a small impact on the surface displacements regardless of whether it is viscoelastic or not. For the same reason, the real parts of the Love numbers vary by only $\sim$10% over this entire range of $\Delta$. This is important because the viscosity profile beneath the very outermost ice layer can perturb the surface stresses only through its effects on the Love numbers, and the figures show that those effects are likely to be no larger than $\sim$10%. Thus there is little additional accuracy to be gained by including a more complicated internal viscosity structure in the shell.

Figs. 1a and 1b also show dips in the values of the imaginary parts of the Love numbers when $\Delta \approx 1$ and $\approx 10^{-5}$, with associated step function increases in the real parts. The $\Delta \approx 1$ features reflect the transition from elastic-like to fluid-like behavior in the outer shell, as $\Delta$ transitions between $<1$ and $>1$. These features result from similar behavior in the imaginary and real parts of $\tilde{\mu}$ and $\tilde{\lambda}$ of the outer shell evident in Figs. 1c and 1d. The features at $\Delta \approx 10^{-5}$ are due to a similar elastic-to-fluid transition in the lower ice layer: a value of $\Delta = 10^{-5}$ in the upper layer implies $\Delta = 1$ in the lower layer because we have chosen to fix the ratio $\eta_{\text{upper}}/\eta_{\text{lower}} = 10^5$.

There are also dips in the imaginary parts of the Love numbers at $\Delta \approx 300$, and corresponding step functions in the real parts. These features, which are far more prominent for the Love number $\tilde{\ell}$ than for $\tilde{h}$, represent the effects of an additional relaxation mode of the system, a mode with a relaxation time that is considerably longer than the Maxwell times of the ice layers. This situation is analogous to the post-glacial-rebound process on the Earth. Viscoelastic models of the Earth exhibit a large suite of relaxation modes. Some directly correspond to Maxwell times. Others, referred to as buoyancy modes, have longer periods and involve