Table 1
Viscoelastic model parameters (Figs. 1, 3, and 4).

Parameter	Symbol	Value
Mass of Europa	M_E	4.8×10^{22} kg
Mass of Jupiter	M_p	1.8987×10^{27} kg
Radius of Europa	R_s	1.561×10^6 m
Europa's surface gravity	g	1.315 m s^{-2}
Europa's orbital semi-major axis	a	6.709×10^8 m
Eccentricity of orbit	ϵ	9.4×10^{-3}
Thickness of stagnant lid	D_{upper}	8×10^3 m
Viscosity of stagnant lid	η_{upper}	10^{22} Pa s
Thickness of convecting ice	D_{lower}	1.2×10^4 m
Viscosity of convecting ice	η_{lower}	10^{17} Pa s
Density of ice	ρ_{ice}	940 kg m^{-3}
Bulk modulus of ice ($= \lambda_{\text{ice}} + \frac{2}{3}\mu_{\text{ice}}$)	κ_{ice}	9.3×10^9 Pa
Shear modulus of ice	μ_{ice}	3.487×10^9 Pa
Thickness of ocean	D_{ocean}	1.5×10^5 m
Density of ocean	ρ_{ocean}	10^3 kg m^{-3}
Bulk modulus of ocean	κ_{ocean}	2×10^9 Pa
Density of silicate interior	ρ_{core}	3.8476×10^3 kg m^{-3}
Bulk modulus of core	κ_{core}	6.67×10^{10} Pa
Shear modulus of core	μ_{core}	4×10^{10} Pa
Complex Love numbers	$\tilde{h}, \tilde{\ell}$	See Tables 2 and 3

Table 2
Diurnal Love numbers for Europa.

Fig.	P_{orbit}	$h_{D,\text{re}}$	$h_{D,\text{im}}$	$\ell_{D,\text{re}}$	$\ell_{D,\text{im}}$
3a, 3b, 3c, 3d	3.55 days	1.192	-6.293×10^{-5}	0.3094	-2.903×10^{-5}
5a	n/a	1.2753	n/a	0.31882 ($\equiv \frac{h}{4}$)	n/a
5b	3.55 days	1.234	0.0	0.3228	0.0

Table 3
NSR Love numbers for Europa.

Fig.	P_{NSR}	$h_{N,\text{re}}$	$h_{N,\text{im}}$	$\ell_{N,\text{re}}$	$\ell_{N,\text{im}}$
4a	1.422×10^5 yr	1.813	-4.186×10^{-3}	0.4748	-2.766×10^{-3}
4b	1.422×10^6 yr	1.834	-2.235×10^{-2}	0.4884	-1.487×10^{-2}
4c	1.422×10^7 yr	1.857	-4.795×10^{-3}	0.5038	-4.883×10^{-3}
4d	1.422×10^8 yr	1.858	-1.334×10^{-3}	0.5095	-1.643×10^{-2}
5c	n/a	1.2753	n/a	0.31882 ($\equiv \frac{h}{4}$)	n/a
5d	6.334×10^7 yr	1.860	-8.597×10^{-4}	0.5047	-2.180×10^{-3}

6.2. NSR tides in the elastic limit ($\Delta \ll 1$)

Next, suppose the NSR period is much shorter than the relaxation timescale of the icy shell, so that $\Delta \ll 1$. In that case $\tilde{\mu} \approx \mu$, and the shell behaves approximately elastically. The shell responds instantaneously to tidal forces, and so the orientation of the tidal bulge is still symmetric about the line to the planet. But now the shell supports shear stresses which try to hold back the bulging fluid ocean, and so the shell's displacement is smaller than in the $\Delta \gg 1$ case. However, as long as the shell is thin (i.e. its thickness is much smaller than the satellite's radius) it has only a minimal effect on the tidal bulge (see Moore and Schubert, 2000; Wu et al., 2001; Wahr et al., 2006; Rappaport et al., 2008), and so the displacements are not affected much by the strength of the shell (i.e. the Love numbers $\tilde{h}$ and $\tilde{\ell}$ are relatively insensitive to the value of Δ). Fixed points in the shell still rise and fall as they nonsynchronously rotate through the bulge, and since $\tilde{\mu}$ is no longer small this periodic motion now results in significant shear stress within the shell. That shear stress is trying (but failing) to confine the ocean. The shear stress is symmetrically distributed about the satellite–planet vector, since the shell is responding instantaneously. The rocky core is still oriented toward the parent planet; it does not participate in the NSR, and so the forcing remains constant at every point in the core. Thus the core still responds to the tidal force as though it were a fluid.

6.3. NSR tides in a truly viscoelastic case ($\Delta \approx 1$)

Finally, consider an intermediate case where the NSR period and the viscous relaxation times are approximately equal (i.e. $\Delta \approx 1$). The icy shell still holds back the ocean slightly since the shell can partially support shear stresses. But because the shell is viscous it does not respond instantaneously to the tidal force. By the time a portion of the shell experiences its maximum displacement, it has rotated slightly beyond the satellite–planet vector. Thus, the bulge in the icy shell is slightly ahead of the satellite–planet vector. The impact of these shear stresses on the shell's displacement is still small, assuming the thickness of the shell is small relative to the radius of the satellite.

Figs. 1a and 1b show how the real and imaginary parts of the Love numbers vary with Δ. Notice that the real parts of the Love numbers are about the same for all values of Δ, and the imaginary parts are always small. Thus the shell's tidal bulge is still closely aligned with the satellite–planet vector. The shell, in effect, still mostly just rides up and down on the underlying ocean during its nonsynchronous rotation. Its tidal displacement field is determined almost entirely by the shape of the ocean surface, and has little dependence on the shell's rheological properties.

The shell's rheology does have a significant impact on the shear stresses caused by that displacement field. Viscoelasticity in the shell can cause the shear stresses to be offset from the satellite–planet vector by up to $\sim 45°$. For example, when $\Delta \approx 1$, $\tilde{\mu}$ and $\tilde{\alpha}$ in Eqs. (32)–(36) have real and imaginary parts that are of the same order, and so they cause $\tilde{\beta}_1$ and the other complex coefficients to have significant imaginary parts even though (for a thin shell) the Love numbers $\tilde{h}$ and $\tilde{\ell}$ are almost real. (Figs. 1c and 1d show how the real and imaginary parts of $\tilde{\mu}$ and $\tilde{\lambda}$ vary with Δ.)

A perhaps counterintuitive result is that the stress pattern ends up being shifted in the direction opposite of the shell's rotation, rather than in the same direction. This can be seen by noting that the real and imaginary parts of Eqs. (32)–(36) have the same sign, for $\tilde{h}$ and $\tilde{\ell}$ real. The stress pattern shifts in this direction because Maxwell viscoelasticity causes the maximum displacement to occur after the maximum stress. Thus since the ocean-imposed displacement field is oriented toward the parent planet, the stress pattern is shifted in the direction the shell has rotated from. Once again, because the core is synchronously rotating, the forcing is constant at every point in the core and so the core still responds as though it were a fluid.

7. Numerical method, and a complication due to the core

For our numerical calculations we consider a satellite composed of four homogeneous compressible layers: a rocky core, an overlying fluid ocean, and a viscoelastic outer icy shell with a stiff upper layer (representing a conductive stagnant lid) and a low-viscosity basal layer (representing a possible convective region).

We recognize that the very cold surface layer of the ice shell will behave as a brittle-plastic material (e.g., Dombard and McKinnon, 2006) rather than viscously even on very long timescales. Whatever effect this near-surface layer has on the surface stresses is thus not accounted for in this model. However, if the fractured layer of the satellite extends to depths at which viscoelastic deformation occurs on the timescale of NSR, then our model should provide an accurate representation of the stresses at that depth.

In this example we choose the ice shell to be thin compared to the satellite's radius, but this is not a requirement of the method; it can be applied equally well to a satellite with a thick outer shell, or even to a satellite with no liquid ocean at all. We determine the diurnal and NSR tidal solutions by solving the equations of motion for a stratified, compressible, and self-gravitating body (Eqs. (6)–(8)). Our numerical method is based on standard