

longer than the Maxwell time, then  $\Delta \gg 1$ , and so  $\tilde{\mu}(\omega) \approx 0$  and  $\tilde{\lambda}(\omega) \approx \lambda + 2\mu/3$  = the elastic bulk modulus. Thus, at very long periods the material cannot support shear stresses, and so behaves as a fluid.

It follows from Eqs. (21) and (22) that  $\tilde{\mu}(-\omega) = \tilde{\mu}^*(\omega)$  and  $\tilde{\lambda}(-\omega) = \tilde{\lambda}^*(\omega)$ , where the superscript \* denotes complex conjugation. Similarly, by using Eqs. (21) and (22) in Eqs. (6)–(8), and taking the complex conjugate of the resulting differential equations, it can be seen that  $\tilde{s}(-\omega) = \tilde{s}^*(\omega)$ , which in turn implies that  $\tilde{h}(-\omega) = \tilde{h}^*(\omega)$  and  $\tilde{\ell}(-\omega) = \tilde{\ell}^*(\omega)$ . Thus, for example, the outer surface radial displacement in response to  $V_{T_0}$  as given in Eq. (20), has the form (using Eq. (9))

$$\begin{aligned} s_r(r = R_s, \theta, \phi, t) &= \frac{Z}{4g} \sin^2 \theta [\tilde{h}(\omega = 2b)e^{i(2\phi+2bt)} + \tilde{h}(\omega = -2b)e^{-i(2\phi+2bt)}] \\ &= \frac{Z}{2g} \sin^2 \theta \operatorname{Re}[\tilde{h}(\omega = 2b)e^{i(2\phi+2bt)}] \\ &= \frac{Z}{2g} \sin^2 \theta [h_{\text{re}}(2b) \cos(2\phi + 2bt) - h_{\text{im}}(2b) \sin(2\phi + 2bt)], \end{aligned} \quad (28)$$

where Re denotes the real part. Note that the imaginary part of  $h$  leads to a displacement component that is out-of-phase with  $V_{T_0}$ . Also note that while the NSR frequency is  $b$ , the relevant forcing frequency in this context is actually  $2b$ , since the NSR potential goes through two complete oscillations over the course of  $360^\circ$  of longitude. Thus  $\Delta_{\text{NSR}} = \mu/(2b\eta)$ . (The factor of 2 in the denominator does not appear in Eqs. (2) and (3) of Harada and Kurita, 2007.)

The surface stresses for a Maxwell rheology are derived from the elastic surface stress results, Eqs. (12)–(14), following the same rationale used for the derivation of Eq. (28). The time-dependent terms are written as the sum of two  $e^{i\omega t}$  terms, Eqs. (24) and (25) are used to replace  $\mu$  and  $\lambda$  when solving the equations of motion, and Eqs. (26) and (27) are used to represent  $h$  and  $\ell$ . Note that both  $\tau_{\theta\theta}$  and  $\tau_{\phi\phi}$  include a time-independent term (the first terms on the right-hand sides of Eqs. (12)–(13)). Since  $\omega = 0$  for these infinite-period terms,  $\Delta$  is infinite, and thus  $\tilde{\mu} = 0$ , so there is no induced stress (since then, after replacing  $\mu$  with  $\tilde{\mu}$ ,  $\beta_1 = \gamma_1 = \beta_2 = \gamma_2 = 0$ ). The remaining terms reduce to

$$\begin{aligned} \tau_{\theta\theta} &= \frac{Z}{2gR_s} \operatorname{Re}[(\tilde{\beta}_1(2b) - \tilde{\gamma}_1(2b) \cos(2\theta))e^{i(2\phi+2bt)} \\ &\quad + 3\epsilon(\tilde{\beta}_1(n) - \tilde{\gamma}_1(n) \cos(2\theta))e^{\text{int}} \cos(2\phi) \\ &\quad - \epsilon(\tilde{\beta}_1(n) + 3\tilde{\gamma}_1(n) \cos(2\theta))e^{\text{int}} \\ &\quad - 4\epsilon(\tilde{\beta}_1(n) - \tilde{\gamma}_1(n) \cos(2\theta))ie^{\text{int}} \sin(2\phi)], \end{aligned} \quad (29)$$

$$\begin{aligned} \tau_{\phi\phi} &= \frac{Z}{2gR_s} \operatorname{Re}[(\tilde{\beta}_2(2b) - \tilde{\gamma}_2(2b) \cos(2\theta))e^{i(2\phi+2bt)} \\ &\quad + 3\epsilon(\tilde{\beta}_2(n) - \tilde{\gamma}_2(n) \cos(2\theta))e^{\text{int}} \cos(2\phi) \\ &\quad - \epsilon(\tilde{\beta}_2(n) + 3\tilde{\gamma}_2(n) \cos(2\theta))e^{\text{int}} \\ &\quad - 4\epsilon(\tilde{\beta}_2(n) - \tilde{\gamma}_2(n) \cos(2\theta))ie^{\text{int}} \sin(2\phi)], \end{aligned} \quad (30)$$

$$\begin{aligned} \tau_{\theta\phi} &= \tau_{\phi\theta} = \frac{2Z}{gR_s} \operatorname{Re}[\tilde{\Gamma}(2b)ie^{i(2\phi+2bt)} \cos \theta \\ &\quad - 4\epsilon \tilde{\Gamma}(n)ie^{\text{int}} \cos \theta \cos(2\phi) - 3\epsilon \tilde{\Gamma}(n)ie^{\text{int}} \cos \theta \sin(2\phi)], \end{aligned} \quad (31)$$

where

$$\tilde{\beta}_1 = \tilde{\mu}[\tilde{\alpha}(\tilde{h} - 3\tilde{\ell}) + 3\tilde{\ell}] = \mu \left[ \frac{\alpha(\tilde{h} - 3\tilde{\ell})}{1 - i\alpha\Delta/3} + \frac{3\tilde{\ell}}{1 - i\Delta} \right], \quad (32)$$

$$\tilde{\gamma}_1 = \tilde{\mu}[\tilde{\alpha}(\tilde{h} - 3\tilde{\ell}) - \tilde{\ell}] = \mu \left[ \frac{\alpha(\tilde{h} - 3\tilde{\ell})}{1 - i\alpha\Delta/3} - \frac{\tilde{\ell}}{1 - i\Delta} \right], \quad (33)$$

$$\tilde{\beta}_2 = \tilde{\mu}[\tilde{\alpha}(\tilde{h} - 3\tilde{\ell}) - 3\tilde{\ell}] = \mu \left[ \frac{\alpha(\tilde{h} - 3\tilde{\ell})}{1 - i\alpha\Delta/3} - \frac{3\tilde{\ell}}{1 - i\Delta} \right], \quad (34)$$

$$\tilde{\gamma}_2 = \tilde{\mu}[\tilde{\alpha}(\tilde{h} - 3\tilde{\ell}) + \tilde{\ell}] = \mu \left[ \frac{\alpha(\tilde{h} - 3\tilde{\ell})}{1 - i\alpha\Delta/3} + \frac{\tilde{\ell}}{1 - i\Delta} \right], \quad (35)$$

$$\tilde{\Gamma} = \tilde{\mu}\tilde{\ell} = \frac{\mu\tilde{\ell}}{1 - i\Delta}. \quad (36)$$

To derive the right-hand sides of Eqs. (32)–(35), we used

$$\tilde{\alpha} = \frac{3\tilde{\lambda} + 2\tilde{\mu}}{\tilde{\lambda} + 2\tilde{\mu}} = \alpha \left( \frac{1 - i\Delta}{1 - i\alpha\Delta/3} \right). \quad (37)$$

There are parameters  $\mu$ ,  $\alpha$ , and  $\Delta$  for each viscoelastic layer. The values for  $\mu$ ,  $\alpha$ , and  $\Delta$  that appear explicitly in Eqs. (32)–(37) are those for the outer surface. Values in other layers (as well as in the surface layer) appear implicitly through their effects on  $\tilde{h}$  and  $\tilde{\ell}$ .

Focusing on just the NSR terms (the terms proportional to  $e^{i(2\phi+2bt)}$  in Eqs. (29)–(31)), since (as we shall see) the NSR stresses are more sensitive to viscoelastic effects than are the diurnal stresses:

$$\begin{aligned} \tau_{\theta\theta} &= \frac{Z}{2gR_s} [(\tilde{\beta}_{1\text{re}}(2b) - \tilde{\gamma}_{1\text{re}}(2b) \cos(2\theta)) \cos(2\phi + 2bt) \\ &\quad - (\tilde{\beta}_{1\text{im}}(2b) - \tilde{\gamma}_{1\text{im}}(2b) \cos(2\theta)) \sin(2\phi + 2bt)], \end{aligned} \quad (38)$$

$$\begin{aligned} \tau_{\phi\phi} &= \frac{Z}{2gR_s} [(\tilde{\beta}_{2\text{re}}(2b) - \tilde{\gamma}_{2\text{re}}(2b) \cos(2\theta)) \cos(2\phi + 2bt) \\ &\quad - (\tilde{\beta}_{2\text{im}}(2b) - \tilde{\gamma}_{2\text{im}}(2b) \cos(2\theta)) \sin(2\phi + 2bt)], \end{aligned} \quad (39)$$

$$\begin{aligned} \tau_{\theta\phi} &= \tau_{\phi\theta} = -\frac{2Z}{gR_s} \cos \theta [\tilde{\Gamma}_{\text{re}}(2b) \sin(2\phi + 2bt) \\ &\quad + \tilde{\Gamma}_{\text{im}}(2b) \cos(2\phi + 2bt)], \end{aligned} \quad (40)$$

where the subscripts re and im denote the real and imaginary parts; and where, for example,  $\tilde{\beta}_{1\text{re}}(2b)$  denotes the real part of  $\tilde{\beta}_1$ , with  $\omega = 2b$  used to evaluate  $\tilde{h}$ ,  $\tilde{\ell}$ , and  $\Delta$ .

## 6. A qualitative description of NSR stresses

To better understand the effects of viscous relaxation on the NSR tidal deformations, we consider the behavior of a satellite in three cases: the fluid limit ( $\Delta \gg 1$ ), the elastic limit ( $\Delta \ll 1$ ), and a truly viscoelastic scenario ( $\Delta \approx 1$ ). For illustrative purposes, we focus on Jupiter's moon Europa, using the parameter values listed in Table 1, and we assume the core is synchronously locked.

### 6.1. NSR tides in the fluid limit ( $\Delta \gg 1$ )

First suppose the NSR period is much longer than the viscoelastic timescale of the floating ice shell, so that  $\Delta \gg 1$ . In that case  $\tilde{\mu}$  is small, and so the shell deforms in response to the NSR tidal forcing nearly as though it were a fluid. Since the rocky core is presumed to be rotating synchronously, it keeps the same face constantly pointing toward the parent planet, and the tidal force at every point in the rocky core never changes: the tidal potential in the rocky core is given by Eqs. (2) and (3) with  $b = 0$ . Since the core's viscosity is presumably finite, the core's shear stresses have had sufficient time to completely relax, and so its tidal response is the same as if it were a perfect fluid. Since the icy shell is assumed to also behave as a fluid in this  $\Delta \gg 1$  limit, and the ocean actually is a fluid, both the icy shell and the rocky core are stretched into ellipsoids oriented toward and away from the parent planet, with amplitudes consistent with the assumption of a completely fluid satellite. Because of the NSR, fixed points in the icy shell are slowly rising and falling as they rotate through the bulge, but there are no induced shear stresses in the shell because  $\tilde{\mu} = 0$  in Eqs. (32)–(36), meaning that  $\tau_{\theta\theta} = \tau_{\phi\phi} = \tau_{\theta\phi} = \tau_{\phi\theta} = 0$  in Eqs. (38)–(40).