balance equation by substituting

$$\rho = \rho_o[1 - \alpha(T - T_o)], \tag{1.10}$$

where ρ_o is the density of the fluid at a reference temperature (T_o) and α is the coefficient of thermal expansion, into equation (1.9). Lithostatic pressure is eliminated from equation (1.9) by substituting

$$P = \rho_o g \hat{e}_z - p, \tag{1.11}$$

where p is the dynamic pressure. With these substitutions, equation (1.9) becomes:

$$\nabla p + \rho_o \alpha (T - T_o) g \hat{e}_z = \nabla \cdot [\eta(\nabla \vec{v} + \nabla^T \vec{v})]. \tag{1.12}$$

It is helpful to notice at this point that the only time dependence in the governing equations appears in the advection-diffusion terms in equation (1.8), and the momentum balance equation (1.12) is time-independent. This occurs because the viscosity of the ice is very large, so thermal diffusion dominates over diffusion of momentum through the fluid by viscous flow. Fluid velocities therefore change very slowly with time. Mathematically, the Navier-Stokes equation (*Kundu*, 1990):

$$\frac{\partial v_i}{\partial t} + v_j \frac{\partial v_i}{\partial x_j} = \frac{-1}{\rho} \frac{\partial p}{\partial x_i} + g \hat{e}_z, \tag{1.13}$$

reduces to

$$v_j \frac{\partial v_i}{\partial x_j} = \frac{-1}{\rho} \frac{\partial p}{\partial x_i} + g \hat{e}_z \tag{1.14}$$

because

$$\frac{\partial v_i}{\partial t} \sim 0, \tag{1.15}$$

and is independent of time. Equation (1.14) is essentially the same as equation (1.12).

In this study, the equations of thermal convection (1.7), (1.8), and (1.12) are solved subject to constant temperature boundary conditions at the surface and base of the fluid layer,

$$T(x, -D) = T_m \tag{1.16}$$

$$T(x, 0) = T_s, \tag{1.17}$$