

$$\pi_2' = \pi_2(d/g) \quad \text{for } d/g \gg 1 \quad (12c)$$

This correction to the vacuum  $\pi_2$  parameter can then be incorporated in Figure 7. Both approaches will be examined after expressing equation (11) in terms of observables.

Two new variables in equation (11) need to be evaluated: ejection velocity,  $v_e$ , and the drag coefficient. Ejecta velocities in equation (11) reflect the impact-induced flow field initiated in the target in response to the shock wave and to first order should be comparable to conditions created by an impact in a vacuum. A simple relation exists between ejection velocity  $v_e$  and crater radius in order to preserve geometric similarity of the ejecta distribution implicitly required for gravity-controlled flow [Post, 1974]. Ejecta thickness at a given distance (scaled to crater radius) from the rim of experimental craters in sand and large lunar craters support this concept. On this basis, a first-order description of the ejection velocity in terms of crater growth can be given [Schultz and Mendell, 1978; Schultz and Gault, 1979]:

$$v_e(x) = k_e(gR_v)^{1/2}(x/R_v)^{-z} \quad (13a)$$

$$v_e = v_o(x/R_v)_c^{-z} \quad (13b)$$

where  $R_v$  is the radius of the crater had it formed in a vacuum as included in equation (10b),  $v_o$  is a minimum ejection velocity, and  $v_e$  is defined as a characteristic late-stage ejection velocity when the transient crater radius  $x$  has achieved a given fraction  $(x/R_v)_c$  of its final size. The exponent  $z$  assumes that crater excavation is an orderly process and can be derived from observations of ejecta velocities through time; computational codes extending to late times [e.g., Schultz et al., 1981]; analytical approximations to the cratering flow field [Maxwell, 1977]; or dimensional analysis [Housen, et al., 1983]. For sand targets at laboratory scales ( $R_v \sim 12$  cm),  $z$  is approximately 2.6 and  $v_o$  approaches 40-70 cm/s [see Schultz and Gault, 1979; Housen et al., 1983]. Here  $(x/R_v)_c$  is taken as  $2/3$  and  $v_o$  is assumed to be 50 cm/s for  $R_v = 8$  cm. Exact values are irrelevant since the purpose is to explore trends over a relatively limited range in scale.

Consequently the effect of air drag in equation (11) can be expressed more simply as

$$d/g \sim C_D \rho R_v / \delta_e g a \quad (14)$$

The drag coefficient  $C_D$  will be a function of the Reynolds number, which is

$$Re = \rho v_e a / \mu \quad (15)$$

where  $\mu$  is the viscosity of the atmosphere (Table 2). At a given Reynolds number,  $C_D$  is constant, whereas at very low Reynolds numbers ( $Re < 1$ ), the drag coefficient becomes  $24/Re$  (Stokes' law). For the adopted reference values and equation (15), ejecta particles with  $v_e$  from a 2 km/s impact into pumice ( $R_v = 8.2$  cm) under one bar atmospheric pressure would correspond to a Reynolds number of about 8. The drag coefficient for Reynolds numbers higher than unity, however,

largely depends on particle shape. Due to such shape effects and the complex interactions within the ejecta curtain, the drag coefficient cannot be predicated *a priori*.

The uncertainty in the drag coefficient requires alternative assumptions in order to constrain the effects of any change. Two approaches are used. First, atmospheric conditions leading to small Reynolds numbers permit the first-order assumption that the drag coefficient is inversely proportional to the Reynolds number. Second, selected data for a very limited range of Reynolds numbers permit assuming a constant value of  $C_D$  and determining directly the effect of air drag (equation (14)) on cratering efficiency corrected for static overpressure.

At small Reynolds numbers, the drag coefficient follows Stokes' law. Equations (11) and (15) then yield the following relation for different conditions:

$$(d/g) = C_D \rho v_e^2 / \delta_e g a \sim (\mu / \rho v_e a) (\rho v_e^2 / \delta_e g a) \quad (16a)$$

$$\text{In general,} \quad (d/g) = \mu v_e / \delta_e g a^2 \quad (16b)$$

or for given gravity and equation (13a),

$$(d/g) \sim (\mu R_v^{1/2}) / \delta_e a^2 \quad (16c)$$

$$\text{or for given target,} \quad (d/g) \sim \mu R_v^{1/2} \quad (16d)$$

or for given target and impact conditions,

$$(d/g) \sim \mu \quad (16e)$$

Equations (16d) and (16e) indicate that the effect of drag for small Reynolds numbers may be masked for a given target but should be clearly exposed for targets with different mean grain sizes (or densities). Figure 8a illustrates the dependence between cratering efficiency corrected for atmospheric pressure (equation (10c)) and the dimensionless drag parameter for four different targets. The additional factor  $\pi_2^{\alpha/3}$  required for consistency with the coupling parameter (equation (10d)) has not been included since the selected data all have similar values of  $\pi_2$ . The scatter for the microsphere data reflect, in part, the effects of crater rim collapse.

Low atmospheric densities and low impact velocities (2.5-2 km/s) minimize any projectile-atmosphere coupling effects and allow the assumption  $C_D \sim 1/Re$  (equation (16c)). The resulting exponent closely matches the  $\pi_2$  scaling exponent,  $-\alpha$ . Consequently, Figure 8b incorporates the alternative approach where drag deceleration replaces gravity in the  $\pi_2$  scaling parameter (equation (12c)) and reveals that the different target types and projectile sizes in Figure 7b are now accommodated by this perspective. Figure 9 further groups a variety of data with similar Reynolds numbers such that  $C_D \sim \text{const}$  (equation (14)) without restriction on atmospheric density and impact velocity. At lower Reynolds numbers and low Mach numbers, the exponent closely matches  $\alpha$  with a dispersion comparable to data under vacuum conditions. As Mach number increases above 10, however, scatter increases. Figure 9b reveals that a high-velocity impact ( $v \sim 6$  km/s) in an argon atmosphere ( $M \sim 18$ ) with  $Re \sim$