where $E(\hat{w})$ is the expected realization of the (normalized) loan portfolio, and the expression in square brackets is the expected repayment by the bank to wholesale creditors, which can be decomposed following Merton (1974) as the repayment made in full in all states of the world minus the option value to default due to the limited liability of the bank. $\pi(\varphi)$ is the value of the put option when the strike price is given by $\varphi = (1+f)L/(1+r)C$.

The contracting problem takes equity E as given and chooses L, C and f to maximize the bank's expected payoff (41) subject to the incentive compatibility constraint for the bank to choose the good portfolio, and the break-even constraint for the wholesale creditors. The incentive compatibility constraint is

$$E_G(\hat{w}) - [\varphi - \pi_G(\varphi)] \geq E_B(\hat{w}) - [\varphi - \pi_B(\varphi)] \tag{42}$$

where $E_G(\hat{w})$ is the expected value of the good portfolio and $\pi_G(\varphi)$ is the value of the put option with strike price φ under the outcome distribution for the good portfolio. $E_B(\hat{w})$ and $\pi_B(\varphi)$ are defined analogously for the expected outcome and option values associated with the bad portfolio. Writing $\Delta\pi(\varphi) = \pi_B(\varphi) - \pi_G(\varphi)$, (42) can be written more simply as

$$\Delta\pi(\varphi) \leq k \tag{43}$$

Incentive compatibility is maintained by keeping leverage low enough that the higher option value to default does not exceed the greater expected payoff of the good portfolio.

Lemma 1 *There is a unique φ^* that solves $\Delta\pi(\varphi) = k$, where $\varphi^* < 1-\varepsilon$.*

The proof is as follows. From Breeden and Litzenberger (1978), the state price density is the second derivative of the option price with respect to its strike price, so that

$$\Delta\pi(\varphi) = \begin{cases} \int_0^{\varphi} F_B(s)\,ds & \text{if } \varphi < 1-\varepsilon \\ \int_0^{1-\varepsilon} F_B(s)\,ds - \int_{1-\varepsilon}^{\varphi} [1 - F_B(s)]\,ds & \text{if } \varphi \geq 1-\varepsilon \end{cases} \tag{44}$$