

capital.

$$\begin{aligned}
p_n &= w + rwn \\
p_{n-1} &= w + rw(n-1) + p_n \\
p_{n-2} &= w + rw(n-2) + p_{n-1} \\
&\vdots \\
p_1 &= w + rw + p_2
\end{aligned} \tag{16}$$

By recursive substitution,

$$\begin{aligned}
p_n &= w(1 + rn) \\
p_{n-1} &= 2w(1 + rn) - wr \\
p_{n-2} &= 3w(1 + rn) - wr(1 + 2) \\
&\vdots \\
p_1 &= nw(1 + rn) - wr(1 + 2 + \dots + (n-1))
\end{aligned} \tag{17}$$

Therefore total sales are

$$\sum_{k=1}^n p_k = w(1 + rn) \left( \sum_{k=1}^n k \right) - wr \sum_{k=1}^{n-1} k(n-k) \tag{18}$$

Using the algebraic identity:

$$\sum_{k=1}^{n-1} k(n-k) = \frac{1}{6}n(n-1)(n+1)$$

total sales in the chain are

$$\sum_{k=1}^n p_k = \frac{1}{2}nw(nr+1)(n+1) - \frac{1}{6}nrw(n-1)(n+1) \tag{19}$$

Total value added in the chain is

$$\begin{aligned}
p_1 &= nw(1 + rn) - wr \sum_{k=1}^{n-1} k \\
&= nw(nr+1) - \frac{1}{2}nrw(n-1)
\end{aligned} \tag{20}$$