The production chain consisting of n firms has output $y(n)$. The output per firm (equivalent to output per worker) is $y(n)/n$. We will adopt the following production function

$$\frac{y(n)}{n} = n^{\alpha}, \qquad (0 < \alpha < 1) \tag{4}$$

The formulation of productivity in our model harks back to Böhm-Bawerk's (1884) notion of "roundabout production", where intermediate goods are used as inputs in further intermediate goods. Our assumption that $0 < \alpha < 1$ captures the feature that:

> "[t]he indirect method entails a sacrifice of time but gains the advantage of an increase in the quantity of the product. Successive prolongations of the roundabout method of production yield further quantitative increases though in diminishing proportion."[3]

The parameter α is the only "deep" technological parameter in our model, as the borrowing rate on working capital will be solved in Section 4 by clearing the credit market. Taking r as given for the moment, we solve for total credit demand, production chain length, and the wage rate. We take the stance of the coalition of firms in maximizing the joint surplus. We may take the solution as the upper bound to any equilibrium solution that incorporates inefficiencies that may arise from incentive problems (see Blanchard and Kremer (1997) and Kim and Shin (2012) for analyses of incentive problems within chains).

The firm coalition's problem at date 0 is to choose n to maximize the expected surplus net of wage costs and all financing costs. Since the borrowing cost is zero until date n and is r from date $n+1$, the firm coalition's problem at date 0 is to choose n to maximize:

$$\sum_{t=n+1}^{\infty} (1-\varepsilon)^{t-n} (n^{\alpha}L - wL - rK) \tag{5}$$

[3]Bohm-Bawerk (1884), p 88 of 1959 English translation by G. Huncke, Libertarian Press.