

At the beginning of date 1, firm  $n$  begins production and sends the intermediate good to firm  $n - 1$  at the end of date 1, who takes delivery and begins production at the beginning of date 2, and so on. Meanwhile, at the beginning of date 2, firm  $n$  starts another sequence of production decisions by producing its output, which is sent to firm  $n - 1$ , and so on.

The first positive cashflow to the chain comes at date  $n + 1$  when firm 1 sells the final output for  $y(n)$ . The cash transfer upstream is instantaneous, so that all upstream firms are paid for their contribution to the output.

Firms borrow by rolling over one period loans. The risk-free interest rate is zero, and is associated with a storage technology that does not depreciate in value. Although the risk-free rate is zero, the firms' borrowing cost will reflect default risk and a risk premium in the credit market. Once the output is marketed from date  $n + 1$ , there is a constant hazard rate  $\varepsilon > 0$  that the chain will fail with zero liquidation value so that lenders suffer full loss on their loans to the chain. Before date  $n + 1$ , there is no probability of failure, and firms can borrow at the risk-free rate of zero. But starting from the loan repayable at date  $n + 1$ , they must borrow at the higher rate  $r > \varepsilon$ , which reflects the default risk  $\varepsilon$  as well as the risk premium, which will be endogenized by introducing a financial sector in Section 4. For now, we treat the borrowing rate  $r$  as given. Firms have limited liability, so that once a production chain fails, the firms in the chain can re-group costlessly to set up another chain of same length by borrowing afresh.

Before the first cash flow materializes to the chain from the sale of the final product, the chain must finance the initial set-up cost of  $\frac{1}{2}n(n + 1)w$ . We can decompose this sum into the steady state inventory  $nw$  that must be carried by the firm in steady state and the initial "triangle" of working capital of  $\frac{1}{2}n(n - 1)w$ . Firms start with no equity and all financing is done by raising debt. Thus, the total initial financing need of the production chain of length  $n$  is given by

$$\frac{n(n + 1)w}{2} \tag{2}$$