intersection of the supply of capital from the agents' precautionary saving and the demand for capital by the economy's firms implied by the marginal product.[23] Crucially, due to the properties of the precautionary savings $E[a]$, the intersection will result in an equilibrium interest rate $r < \rho$ which means that the steady state level of capital violates the modified golden rule level given by

$$f'(k^*) - \delta = \rho. \tag{27}$$

This rule requires that the rate at which consumption today can be exchanged against consumption tomorrow given the economy's technology should equal the rate at which agents trade off consumption today against consumption tomorrow. Given the technology in the present economy, one unit of consumption today can instead be used as capital which yields $f'(k)$ in extra output tomorrow and leaves $1 - \delta$ units of capital that can be consumed. Therefore one unit of consumption today can be exchanged for $1 + f'(k) - \delta$ units of consumption tomorrow. Given the agents' preferences in the present economy, they are willing to exchange one unit of consumption today for $1 + \rho$ units of consumption tomorrow. For the two rates to be the same, capital has to be at the level k^* given by equation (27). The individual agent's precautionary saving motivated by the uninsured risk and constrained borrowing however leads to an excessively high level of aggregate savings $k > k^*$ that is socially wasteful.

In a slightly modified framework, Aiyagari (1995) shows how a tax on capital earnings can address the violation of the modified golden rule. Such a tax works by driving a wedge between the gross interest rate r that capital earns based on its marginal product and the net interest rate $\bar{r}$ agents receive and adjust their asset holdings to. As pointed out by Aiyagari, simply crowding out the excessive investment by issuing government debt paying the same return as capital does not work. Since the precautionary saving diverges as the interest rate approaches the discount rate no finite amount of government debt can achieve $r = \rho$. This is a significant difference to the OLG literature and the model by Woodford (1990) discussed above. However, this argument relies on transfers in the form of government spending on public goods and it does not address the potential of improving risk sharing among agents.

Angeletos (2007) studies a model analogous to Aiyagari's but assumes that the idiosyncratic shocks are to capital income instead of labor income. In this case the

---

[23]Note that the supply of capital $E[a]$ also depends on the wage which can be expressed as a function of r since $w = f(k) - kf'(k)$.