$-u'''/u'' > 0$, see Kimball (1990).[19]

Instead of assuming convexity of u' we can impose a borrowing constraint $a_t \geq -b$ for some exogenous borrowing limit $b > 0$. With the borrowing constraint, the Euler equation (25) changes to

$$u'(c_t) \geq \beta (1 + r) E_t [u'(c_{t+1})], \tag{26}$$

with equality if $a_{t+1} > -b$. With a borrowing constraint, marginal utility can only be equalized as long as the constraint is not binding. When the constraint is binding, the marginal value of transferring one unit of consumption from period $t + 1$ to period t is positive but cannot be accomplished.

If we define a new variable $M_t = \beta^t (1 + r)^t u'(c_t)$ then we have $M_t \geq 0$ and we can rewrite the Euler equation (26) as

$$M_t \geq E_t [M_{t+1}].$$

This implies that M_t is a bounded supermartingale so we can make use of Doob's convergence theorem. From the definition of M_t we see that the crucial role for the convergence is played by $\beta (1 + r) \lessgtr 1$. If the agent is relatively patient given the interest rate, i.e. $\beta (1 + r) > 1$, then convergence of M_t requires $u'(c_t)$ to go to zero. This means that the agent's consumption c_t goes to infinity and this can only be achieved if the asset holdings a_t also go to infinity. The same can be shown to hold for the borderline case of $\beta (1 + r) = 1$, see Chamberlain and Wilson (2000) for details. Only in the case $\beta (1 + r) < 1$, where the agent is relatively impatient, will consumption c_t and therefore asset holdings a_t not diverge.

To illustrate the precautionary saving in this setting it is important to highlight the difference to the case without uncertainty, where the Euler equation given by

$$u'(c_t) \geq \beta (1 + r) u'(c_{t+1})$$

with equality if $a_{t+1} > -b$. For $\beta (1 + r) > 1$ the agent would also accumulate an infinite amount of assets as in the case with uncertainty in (26) while for the borderline case $\beta (1 + r) = 1$ the agent would maintain any initial asset holdings. For $\beta (1 + r) <$

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[19]While the initial work emphasized consumption smoothing, e.g. Hall (1978), there is a large literature on precautionary saving of individual agents in this tradition, see Zeldes (1989), Caballero (1990, 1991), Deaton (1991), Carroll and Kimball (1996), Carroll (1997).