4.2 Precautionary Savings and Uninsurable Idiosyncratic Risk

Agents with a dislike for fluctuations in consumption over time face a problem if their income stream is not steady. Anticipated fluctuations in income create a demand for consumption smoothing, which requires saving in periods with high income and borrowing in periods with low income. If markets are incomplete so agents cannot insure against uncertain fluctuations in income then an additional precautionary motive for saving can arise.

4.2.1 Precautionary Savings

There are two ways to model a precautionary motive for saving, through special assumptions on the shape of the utility function or through a borrowing constraint.

Consider an agent who maximizes

$$E_0\left[\sum_{t=0}^{\infty}\beta^t u\left(c_t\right)\right]$$

subject to the budget constraint

$$c_t + a_{t+1} = e_t + (1+r)\, a_t \quad \text{for all } t, \tag{24}$$

where e_t is the potentially random endowment in period t and a_{t+1} are the assets held from period t to period $t+1$. The standard Euler equation for this problem is given by

$$u'\left(c_t\right) = \beta\left(1+r\right) E_t\left[u'\left(c_{t+1}\right)\right]. \tag{25}$$

If we assume that the marginal utility u' is convex, i.e. $u''' > 0$, then Jensen's inequality implies

$$\frac{E_t\left[u'\left(c_{t+1}\right)\right]}{u'\left(c_t\right)} > \frac{u'\left(E_t\left[c_{t+1}\right]\right)}{u'\left(c_t\right)}$$

so the marginal value of transferring one unit of consumption from period t to period $t+1$ is greater if consumption in period $t+1$ is variable. Therefore the optimal level of consumption in period t will be lower with uncertainty than without, the difference being precautionary saving. This notion of precautionary saving is typically referred to as "prudence" and can be measured similar to risk aversion by a prudence coefficient