

The second setup highlights the possibility of crowding-in. Here the two types of agents have alternating opportunities to invest in capital. The unproductive agents can only hold government debt while the productive agents can hold capital and government debt with potentially different returns  $f'(k)$  and  $1 + r$ , respectively. Woodford then studies a stationary equilibrium where the unproductive agents are unconstrained, consume  $\bar{c}$  and save part of their endowment in government debt while the productive agents are constrained, invest their savings and part of their endowment in capital and consume  $\underline{c} \leq \bar{c}$ . In this equilibrium the Euler equations for the unconstrained and the constrained agent, respectively are

$$\begin{aligned} u'(\bar{c}) &= \beta(1+r)u'(\underline{c}), \\ u'(\underline{c}) &= \beta f'(k)u'(\bar{c}), \end{aligned}$$

while the interest rate satisfies  $1 + r \leq f'(k)$ .

Combining the two Euler equations we now have  $\beta(1+r) = (\beta f'(k))^{-1}$ . While an increase in government debt still increases the interest rate  $r$ , this now leads to an *increase* in the level of capital  $k$ . The additional liquidity allows the agents to transfer more wealth from unproductive periods to productive periods and therefore increases the investment in capital. To achieve efficiency the government should again increase its debt until the borrowing constraint doesn't bind anymore.

A similar crowding-in effect of bubbles is illustrated in [Martin and Ventura \(2010\)](#) where entrepreneurs are constrained to borrowing a fraction of their future firm value. While efficiency requires that all investment should be undertaken by firms with high investment productivity, the borrowing constraint restricts the flow of funds to these firms. The paper then analyses the effect of rational bubbles on firm values. As in [Tirole \(1985\)](#) discussed above, the bubbles crowd out total investment since they use up part of savinds. In the present setting, however, a bubble also relaxes the borrowing constraint of firms with investment opportunities which improves the allocation of funds to the productive firms and *crowds in* their investment. This increase in allocation efficiency outweighs the effect of lower aggregate investment and the bubbles are possible even if the economy is dynamically efficient, as long as there is a borrowing friction.