

can only exist in a dynamically inefficient economy since otherwise any positive bubble would eventually outgrow the economy.

**Crowding out or Crowding in?** Woodford (1990) shows that instead of the standard crowding-out effect, government debt can also have a *crowding-in effect*, which increases investment. He studies the effect of borrowing constraints in an economy of two types of agents with either time-varying endowments or time-varying investment opportunities. There are no aggregate fluctuations since the two agent's individual fluctuations are perfectly negatively correlated and deterministic. Nevertheless, in the presence of borrowing constraints the agents can only transfer wealth forward in time which creates a demand for a store of value. Woodford assumes that agent's cannot borrow at all and can save by holding capital and government debt which both pay interest  $r$ .

The paper studies two setups, each with two types of infinitely-lived agents in an economy with per-capita production function  $f(k)$ . In the first setup, the two types of agents have alternating endowments  $\bar{e} > \underline{e} \geq 0$ . Woodford studies a stationary equilibrium where in each period, agents with high endowment  $\bar{e}$  are unconstrained, consume  $\bar{c}$  and save part of their endowment while the agents with low endowment  $\underline{e}$  are constrained and consume their endowment and savings for a total consumption  $\underline{c} \leq \bar{c}$ . In this equilibrium the Euler equations for an unconstrained and a constrained agent, respectively are

$$\begin{aligned} u'(\bar{c}) &= \beta(1+r)u'(\underline{c}), \\ u'(\underline{c}) &\geq \beta(1+r)u'(\bar{c}), \end{aligned}$$

while the interest rate satisfies  $1+r = f'(k)$ .

Combining the two Euler equations we see that in this equilibrium we have  $\beta(1+r) \leq 1$  or  $r \leq \rho$ , i.e. the interest rate is lower than the agents' discount rate so they are relatively impatient and therefore the borrowing constraint is binding. If the government increases the amount of debt outstanding it provides additional liquidity for agents saving which increases the interest rate and therefore reduces the capital stock. This mechanism is the same as the classic crowding-out effect of government debt in the OLG models discussed above. In Woodford's model the government can increase its debt sufficiently to achieve efficiency with  $r = \rho$ , where the borrowing constraint doesn't bind anymore and we have  $\underline{c} = \bar{c}$ .